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# Wavelet Galerkin Methods for Aerosol Dynamic Equations in Atmospheric Environment

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**Abstract.** Aerosol modelling is very important to study and simulate the behavior of aerosol dynamics in atmospheric environment. In this paper, we consider the general nonlinear aerosol dynamic equations which describe the evolution of the aerosol distribution. Continuous time and discrete time wavelet Galerkin methods are proposed for solving this problem. By using the Schauder's fixed point theorem and the variational technique, the global existence and uniqueness of solution of continuous time wavelet numerical methods are established for the nonlinear aerosol dynamics with sufficiently smooth initial conditions. Optimal error estimates are obtained for both continuous and discrete time wavelet Galerkin schemes. Numerical examples are given to show the efficiency of the wavelet technique.

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**Key words**: Aerosol dynamics, wavelet Galerkin methods, semi-discrete, fully-discrete, existence, error estimate.

## 1 Introduction

The distribution of aerosol particles in atmospheric environment has been recognized to be of significance due to their effects on climate change and human health. Aerosol modeling has been playing an important role in studying and simulating the behavior of aerosol dynamics in atmosphere. The aerosol dynamic equations in terms of the aerosol size distribution function describe different processes evolved in the lifetime of aerosols, which include condensation, nucleation, coagulation, and deposition. The equations are

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nonlinear differential and integral equations. Some numerical methods have been studied to solve the equations such as sectional method [21], moment method [5, 32], modal method [2, 36], finite element method [31], and stochastic approach [17], etc. The conventional sectional approach has some limitation such as numerical diffusion and lower accuracy, while the modal approach has the high numerical efficiency but less physical representation of real aerosol distribution and overlap of various models [35]. But, on the other hand, there have been few works on the theoretical analysis of numerical methods to the nonlinear aerosol dynamics which will definitely indicate to develop efficient numerical techniques for the problem.

Wavelet multiresolution analysis was originally applied as a powerful tool for signal and image processing. Wavelets cut up data into different frequency components and then study each component with a resolution matched to its scale. The wavelet technique has great advantage of approximating the signals which contain discontinuities and sharp spikes. Recently, wavelet techniques have been applied to many areas in applied mathematics including developing numerical schemes to solve the integral equations and the partial differential equations (see, e.g., [1, 7, 8, 10, 11, 13–15, 23, 24, 29, 33]). Papers [10, 15, 29] developed efficient multilevel wavelet methods for solving nonlinear integral equations on bounded domains. [7, 13] developed wavelet Galerkin methods for the numerical solution of second-order elliptic equations. Paper [1] considered wavelet Galerkin methods for solving quasilinear hyperbolic conservation equations. [14] studied generalized Petrov-Galerkin schemes with multiscale techniques for solving pseudodifferential equations. [33] studied wavelet methods for parabolic problems combining with a strongly elliptic pseudodifferential operator. Recently, paper [24] applied the wavelet collocation method to compute the population balance equation. However, there is no theoretical analysis provided for the methods to this problem.

Due to the localization properties that wavelets display both in space and frequency, the wavelet multiresolution analysis allows us to obtain an efficient sparse representation of the solution function, especially useful when the solution contains singularities, irregular structure and transient phenomena. Wavelet methods can distinguish smooth and singular regions automatically. This leads to the most important advantage that the wavelets can efficiently and accurately approximate sharp changes of solution functions. On the other aspect, due to different condensation, coagulation and removal mechanisms, the aerosol size distribution is highly uneven distributed, such as multiple lognormal distributions in some regions. Thus, it is very important to efficiently solve the aerosol dynamic equations in size and time where the aerosol distributions vary very sharply in the size direction. This motivates us to develop and analyze wavelet Galerkin methods for solving the aerosol dynamic equations in this paper.

In this paper, we propose and analyze wavelet Galerkin methods for solving the nonlinear aerosol dynamic equations. The aerosol dynamic equations are nonlinear differential and integral equations which contain a nonlinear Volterra type integral term and a nonlinear Fredholm type integral term of the size distribution function. We introduce wavelet Galerkin methods to construct both the semi-discrete scheme and the