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Operator Splitting for Three-Phase Flow in Heterogeneous Porous Media

E. Abreu[1], J. Douglas[2,*], F. Furtado[3] and F. Pereira[4]

[1] *Instituto Nacional de Matemática Pura e Aplicada, RJ 22460-320, Brazil.*
[2] *Department of Mathematics, Purdue University, West Lafayette, IN 47907-1395, USA.*
[3] *Department of Mathematics, University of Wyoming, Laramie, WY 82071-3036, USA.*
[4] *Department of Mathematics and School of Energy Resources, University of Wyoming, Laramie, WY 82071-3036, USA.*

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Abstract. We describe an operator splitting technique based on physics rather than on dimension for the numerical solution of a nonlinear system of partial differential equations which models three-phase flow through heterogeneous porous media. The model for three-phase flow considered in this work takes into account capillary forces, general relations for the relative permeability functions and variable porosity and permeability fields. In our numerical procedure a high resolution, nonoscillatory, second order, conservative central difference scheme is used for the approximation of the nonlinear system of hyperbolic conservation laws modeling the convective transport of the fluid phases. This scheme is combined with locally conservative mixed finite elements for the numerical solution of the parabolic and elliptic problems associated with the diffusive transport of fluid phases and the pressure-velocity problem. This numerical procedure has been used to investigate the existence and stability of nonclassical shock waves (called transitional or undercompressive shock waves) in two-dimensional heterogeneous flows, thereby extending previous results for one-dimensional flow problems. Numerical experiments indicate that the operator splitting technique discussed here leads to computational efficiency and accurate numerical results.

AMS subject classifications: 76S05, 76T30, 78M10, 78M20

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1 Introduction

The study of operator splitting techniques has a long history and has been pursued with various methods. Since alternating-direction methods were introduced by Douglas,

*Corresponding author. *Email addresses:* `eabreu@impa.br` (E. Abreu), `douglas@math.purdue.edu` (J. Douglas), `furtado@uwyo.edu` (F. Furtado), `lpereira@uwyo.edu` (F. Pereira)

Peaceman and Rachford [1–5] and fractional step methods by D'jakonov, Marchuk and Yanenko [6, 7], these procedures, which reduce the time-stepping of multidimensional problems to locally one-dimensional computations, have been applied in the numerical simulation of many physically important problems, including reservoir flow problems, particularly in the case of single and two-phase flows. Here, the operator splitting is based on separating the underlying physical processes and treating each such process appropriately; thus, instead of solving the governing differential equations in the form which results directly from the basic conservation laws (supplemented by constitutive relations), the system of equations will be rewritten in such a way as to exhibit clearly each physical process. Then, distinct, appropriate numerical techniques can be orchestrated within an operator-splitting formulation to furnish effective and efficient numerical procedures designed to resolve the sharp gradients and dynamics evolving at vastly different rates which are the hallmarks of reservoir flow problems.

We present an operator splitting technique for the numerical solution of a highly nonlinear system of differential equations modeling three-phase flow through heterogeneous porous media. Three-phase flow in porous media is important in a number of scientific and technological contexts, including enhanced oil recovery [8–14], geological CO_2 sequestration [15], and radionuclide migration from repositories of nuclear waste [16, 17].

We consider the governing system of equations written in terms of the oil pressure (see, e.g., [18, 19]); this formulation allows us to identify a subsystem of nonlinear hyperbolic conservation laws (associated with convective transport), a parabolic subsystem of equations (associated with diffusive transport), and a elliptic subsystem (associated with the pressure-velocity calculation). Our splitting procedure solves the elliptic, hyperbolic, and parabolic subsystems sequentially, using numerical methods specifically tailored to such types of partial differential equations. We remark that it would have been very difficult, if not impossible, to employ such state-of-the-art numerical schemes had we attempted to solve the original system by standard implicit procedures. Moreover, any implicit procedure would require considerably more expensive computations since large linear and nonlinear problems, which do not appear in the splitting scheme, would have to be treated. Our splitting technique allows time steps for the pressure-velocity calculation that are longer than those for the diffusive calculation, which, in turn, can be longer than those for convection.

For three-phase flow, distinct empirical models have been proposed for the relative permeability functions [20–22], and more recently [23]. In addition, it is well known that for some of these models [20–22], which have been used extensively in petroleum engineering, the 2×2 system of conservation laws (the saturation equations) that arises when capillarity (diffusive) effects are neglected fails to be strictly hyperbolic somewhere in the interior of the saturation triangle (the phase space). This loss of strict hyperbolicity frequently leads to the occurrence of nonclassical shock waves (called transitional or undercompressive shock waves) in the solutions of the three-phase flow model. Crucial to calculating transitional shock waves is the correct modeling of capillarity effects [24]. Thus, their accurate computation constitutes a *bona fide* test for numerical simulators.