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An Interface-Capturing Method for Resolving Compressible Two-Fluid Flows with General Equation of State

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Abstract. In this study, a stable and robust interface-capturing method is developed to resolve inviscid, compressible two-fluid flows with general equation of state (EOS). The governing equations consist of mass conservation equation for each fluid, momentum and energy equations for mixture and an advection equation for volume fraction of one fluid component. Assumption of pressure equilibrium across an interface is used to close the model system. MUSCL-Hancock scheme is extended to construct input states for Riemann problems, whose solutions are calculated using generalized HLLC approximate Riemann solver. Adaptive mesh refinement (AMR) capability is built into hydrodynamic code. The resulting method has some advantages. First, it is very stable and robust, as the advection equation is handled properly. Second, general equation of state can model more materials than simple EOSs such as ideal and stiffened gas EOSs for example. In addition, AMR enables us to properly resolve flow features at disparate scales. Finally, this method is quite simple, time-efficient and easy to implement.

AMS subject classifications: 35L65, 65M50, 65M99, 76T99

Key words: MUSCL-Hancock scheme, adaptive mesh refinement, compressible two-fluid flows, general equation of state.

1 Introduction

Numerical simulation of compressible multi-fluid flows has attracted much attention during the past few decades because they widely exist in nature and industrial applications. However, direct application of existing high-resolution methods such as Godunov-type and WENO schemes to these flows gives rise to unphysical pressure oscillations on

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material interfaces. This problem was solved by Abgrall by assuming pressure equilibrium across a material interface [2]. Since then, much progress has been made by following the work of Abgrall, refer to [3,8,13–17,23,25,26], etc. All these methods including the one developed in this study are based on volume fraction formulation and belong to the interface-capturing family. In other words, the interfaces are not tracked accurately, but allowed to diffuse numerically over a few grid cells. The fluids are marked by volume fraction of one component. In transition layers between two components, value of volume fraction lies between 0 and 1 and a mixture or artificial equation of state (EOS) needs to be introduced to model the mixture. Compared with other methods like interface-tracking, volume of fluid, and level-set methods, the interface-capturing method is easier to implement but less accurate. The interface-tracking can accurately track the material interfaces without introducing numerical diffusion. The volume of fluid reconstructs the interfaces using complicated interpolation schemes. Although accurate, the two methods will be very complex when applied to three-dimensional problems and cases with drastic topological changes. The level-set method is simple but not discretely conservative. The present interface-capturing method is less accurate as it may admit excessive numerical diffusion on the interfaces. However, we can use adaptive mesh refinement (AMR) to improve resolution. This method can be easily applied to three-dimensional problems and those with complex EOS. See [3, 8] and references cited therein for comparison of different methods.

Although much research has been devoted to resolution of multi-fluid flows, few studies have been done on fluids modeled by general or complex EOS. In this study, we concentrate on extension of MUSCL-Hancock scheme, HLLC Riemann solver and AMR to treatment of flows with Mie-Grüneisen EOS, which includes cases of stiffened gas, van der Waals, Jones-Wilkins-Lee EOSs, etc. These EOSs can model a wide range of real materials in practice, but they also introduce difficulties into numerical algorithm. In our method, mass of each fluid is conserved because material parameters in the mixture EOS depend on the partial densities. However, only the momentum and energy equations for the mixture are included. The material interfaces are identified using the volume fraction, which obeys an advection equation. Physically, the volume fraction is passively advected at local speed. Nevertheless, some of methods in the literature update value of the volume fraction according to the velocity field at previous time step. This, however, can lead to unphysical partial densities and failure of calculations under certain situations. To rectify this problem, in this study, the modified HLLC Riemann solver developed in [8] is generalized to construct numerical fluxes, particularly those of the advection equation. Since these fluxes are constructed using solutions of Riemann problems and information of the velocity field at present time step has been taken into account, the volume fraction is advected properly. The resulting scheme is stabler than others. In addition, HLLC is less expensive computationally than other algorithms like two-shock solver where iteration is needed in finding solution, and therefore, our method is time-efficient. In this study, MUSCL-Hancock scheme is extended to construct input states for the Riemann problems. This scheme with minmod limiter proves to be very robust and easy to imple-