

The Riemann Problem for the Dispersive Nonlinear Shallow Water Equations

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Received 24 November 2008; Accepted (in revised version) 30 March 2009

Communicated by Chi-Wang Shu

Available online 16 July 2009

Abstract. The complete analytical solution of the Riemann problem for the homogeneous Dispersive Nonlinear Shallow Water Equations [Antuono, Liapidevskii and Brocchini, *Stud. Appl. Math.*, 122 (2009), pp. 1-28] is presented, for both wet-bed and dry-bed conditions. Moreover, such a set of hyperbolic and dispersive depth-averaged equations shows an interesting resonance phenomenon in the wave pattern of the solution and we define conditions for the occurrence of resonance and present an algorithm to capture it. As an indirect check on the analytical solution we have carried out a detailed comparison with the numerical solution of the government equations obtained from a dissipative method that does not make explicit use of the solution of the local Riemann problem.

PACS: 47.11.-j, 02.70.-c, 47.35.-i

Key words: Riemann problem, Dispersive Nonlinear Shallow Water Equations, compound waves, resonance.

1 Introduction: The Dispersive Nonlinear Shallow Water Equations

The most popular approximate model equations for studying nearshore hydrodynamics are the Nonlinear Shallow Water Equations (NSWE) and many available Boussinesq type equations (BTEs), which all stem from the work of Peregrine [12]. BTEs are capable to model dispersive effects and are valid throughout a wide portion of the nearshore

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zone, but they cannot directly account for wave breaking and they cannot intrinsically predict the motion or the position of the shoreline [3]. On the contrary, the classical Non-linear Shallow Water Equations allow for a simple treatment of wave breaking and of the shoreline motion, but they cannot model dispersive effects and their validity is limited to a narrow area close to the shore. In order to combine the advantages of these models, Antuono, Liapidevskii and Brocchini [1] proposed a new set of depth-averaged equations, called Dispersive Nonlinear Shallow Water Equations (DNSWE), which are dispersive and hyperbolic at the same time. These equations, obtained by using a hyperbolic approximation of a chosen set of nonlinear and weakly-dispersive Boussinesq-type equations, provide both a physically sound description of the nearshore dynamics and a complete representation of dispersive and nonlinear wave phenomena. A detailed description of the conditioning of the dispersive terms and of the related hyperbolic approximation can be found in Antuono, Liapidevskii & Brocchini [1]. Here, a complete description of the main advantages of the DNSWE is also given.

The 1D set of dimensional DNSWE can be written in the following conservative form:

$$\begin{cases} d_t + Q_x = 0, \\ \left[Q \left(1 - \frac{h_{xx}}{6} d \right) \right]_t + \left(\frac{g d^2}{2} + \frac{Q^2}{d} + \frac{A d^2}{3} \psi - \frac{g h_x^2}{6} d^2 \right)_x \\ = \frac{A h_x}{3} d \psi - g \frac{h_x^3}{3} d - g \frac{h_x h_{xx}}{3} d^2 + g h_x d, \\ \phi_t = \psi, \\ \gamma \psi_t + Q_x = -A \phi, \end{cases} \quad (1.1)$$

where $Q=ud$ is the flow rate, A is a positive dimensional parameter ($[A]=T^{-2}$) generally set to $1s^{-2}$, γ is a positive dimensionless parameter ($\gamma \ll 1$) and ϕ and ψ are two potential functions. As shown in Fig. 1, h is the still water level, u is the onshore velocity and d is the total water depth.

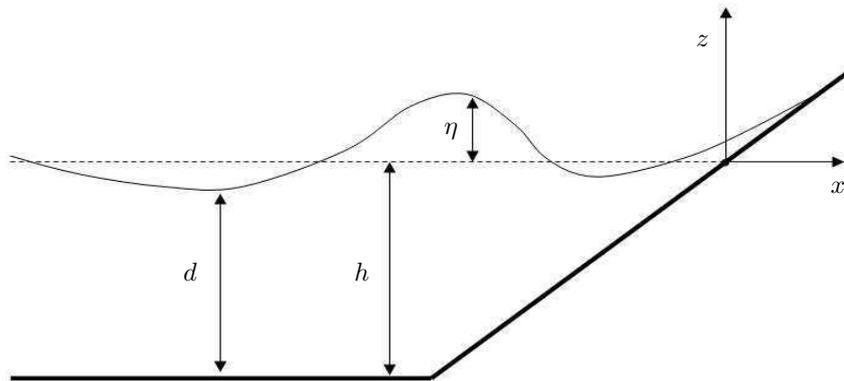


Figure 1: The reference frame of the DNSWE.