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A Lumped Particle Modeling Framework for Simulating Particle Transport in Fluids

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Abstract. This paper presents a lumped particle model for simulating a large number of particles. The lumped particle model is a flexible framework in modeling particle flows, embodying fundamental features that are intrinsic in particle laden flow, including advection, diffusion and dispersion. In this paper, the particles obey a simplified version of the Bassinet-Boussinesq-Oseen equation for a single spherical particle. However, instead of tracking the individual dynamics of each particle, a weighted spatial averaging procedure is used where the external forces are applied to a "lump" of particles, from which an average position and velocity is derived. The temporal evolution of the particles is computed by partitioning the lumped particle into smaller entities, which are then transported throughout the physical domain. These smaller entities recombine into new particle lumps at their target destinations. For particles prone to the effects of Brownian motion or similar phenomena, a symmetric spreading of the particles is included as well. Numerical experiments show that the lumped particle model reproduces the effects of Brownian diffusion and uniform particle transport by a fluid and gravity. The late time scale diffusive nature of particle motion is also reproduced.

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1 Introduction

Consider sand and mud particles suspended in seawater, being moved around by the fluid flow before they eventually settle on the sea floor. This process is at the heart of

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computational studies of how sedimentary rocks are formed by erosion and deposition. In geoscience, such studies are referred to as depositional modeling [27]. When simulating the flow of particle-laden fluids, many physical aspects must be considered; the frequency of interparticle collisions, the ambient flow configuration including the degree of turbulence, and gravitational effects, to name a few. The size of the suspended particles is also an important factor, as particles of micrometers or less will have Brownian motion as well. For instance, all of the above mentioned effects must be taken into account when studying *turbidity currents* [41], which take the form of a highly turbulent sand-laden subaquatic flow. Turbidity currents are often triggered by tsunamis, earthquakes, or underwater avalanches, and contribute significantly to the transport of sediments into deep marine areas [25].

The accurate simulation of particle transport and the correct modeling of turbulence are two central aspects in understanding turbidity currents. This paper will focus on the simulation of the particles in the flow.

Although there is great variation in the details, the simulation of particle flows can roughly be categorized into two distinct approaches: *discrete particle models* and *Eulerian continuum models*. In discrete particle models, the motion of either single particles or clusters of particles are simulated individually by applying forces as prescribed by Newton's laws. The Eulerian models, however, treat the particles as a continuum where averaged equations of motions are solved instead.

The most common of the discrete particle methods is the Lagrangian approach [23]. Here, each particle's position and velocity are obtained by integrating an equation describing the particle's motion, which is usually the Bassinet-Boussinesq-Oseen (BBO) equation [36]. This equation is coupled with variants of the Navier-Stokes equation to obtain a description of the particle flow [30]. Usually, this coupling is one-way, meaning that the particles do not influence the ambient fluid. However, two-way couplings are often needed for dense particle flows, for which the particles' effect on the fluid is modeled by either including body force terms in the Navier-Stokes equations or adding a particle density dependent fluid viscosity. The Lagrangian approach has also the advantage of being applicable to problems covering a wide range of Reynolds numbers [24, 38]. More details on Lagrangian models are available in [5] and the references therein.

For low Reynolds numbers, a widely used discrete particle method is the *Stokesian dynamics* approach [28]. Here, the linearized hydrodynamic equations are solved for the particles and the fluid [18], where both the rotational and translational aspects are studied. Forces and torques on the particles are obtained by integrating over the particles' surface, the net effect of which is then applied to the surrounding fluid as well. Stokesian dynamics have been very successful in reproducing observed results, such as Brownian motion, in addition to correctly predicting values for drag coefficients However, this method is computationally very expensive for dynamically evolving systems, since the computational effort scales as the cube of the number of particles [19]. Currently, simulations based on this method is limited to 100 or less. Stokesian dynamics is, however, a useful tool to study basic particle physics at low Reynolds numbers.