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A Spectrally Accurate Boundary Integral Method for Interfacial Velocities in Two-Dimensional Stokes Flow

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Abstract. We present a new numerical method for solving two-dimensional Stokes flow with deformable interfaces such as dynamics of suspended drops or bubbles. The method is based on a boundary integral formulation for the interfacial velocity and is spectrally accurate in space. We analyze the singular behavior of the integrals (single-layer and double-layer integrals) appearing in the equations. The interfaces are formulated in the tangent angle and arc-length coordinates and, to reduce the stiffness of the evolution equation, the marker points are evenly distributed in arc-length by choosing a proper tangential velocity along the interfaces. Examples of Stokes flow with bubbles are provided to demonstrate the accuracy and effectiveness of the numerical method.

AMS subject classifications: 45F15, 65R20, 76T10

Key words: Boundary integral method, Stokes flow, two-phase flow, weakly singular integral, spectral accuracy.

1 Introduction

Stokes flows involving interfaces have been studied extensively in the past a few decades. It has many important applications in science and engineering, such as in biomechanics, geophysics, mechanical engineering, and chemical engineering. Numerical solutions can accurately address practical questions when analytical solutions cannot be found and real experiments are hard to realize or expensive to execute. Computer simulations have become a very important tool in studying interfacial dynamics in low Reynolds number flow.

There are many numerical methods that are suitable for computing interfacial dynamics in Stokes flow, which can be divided into two categories. Sharp interface modeling, where the interface separating two fluids has zero thickness, include boundary integral methods, level set methods, immersed interface methods, volume-of-fluid and front

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tracking methods among others (see [5, 8, 10, 14, 16] and the references therein). Other methods represent interfaces using a thin transition layer, such as immersed boundary methods [11], phase-field or diffused interface methods [2]. Each method has its merits and disadvantages. Location and shape of interfaces are accurately represented by boundary integral methods but topological changes are difficult to handle for this kind of interface tracking methods. In this work, we focus on the boundary integral methods to solve the interfacial dynamics problem in two-dimensional Stokes flow. The three-dimensional problem is more challenging to study especially for evolving interfaces with large deformations.

Among boundary integral methods, boundary element method is probably the most popular one in practical applications [12]. In this method, an interface is represented by a union of boundary elements. It is reliable and computationally efficient. However, it usually suffers from low order of accuracy in space, depending on the order of the elements and the quadrature employed to compute the integrals. Quadratic elements like parabolic and circular arc elements are commonly used in practice, resulting at most second-order accuracy in space. Spectral element methods [3] use higher order orthogonal polynomials on each boundary element to achieve spectral accuracy with respect to numerical integration on individual elements. However, the spectral accuracy over the entire interface is hard to achieve using spectral element methods due to geometric discontinuities at the edges of the elements. Dimitrakopoulos and Wang [3] developed a suitable interfacial smoothing based on Hermitian-like interpolations to maintain the continuities of the interface at the edges of the spectral elements. Recently, Kropinski [7] presented a boundary integral method using Fourier series to represent the interface and improved the spatial accuracy. In [7], the two-dimensional Stokes equations are reformulated based on theory of complex variables. Kropinski [7] solves the Sherman-Lauricella integral equation for a complex density function defined on the interface. The interfacial velocity is then obtained by evaluating boundary integrals of the complex density. We present a boundary integral method that also uses the Fourier representations of the interfaces but solves the velocity on the interface directly from the boundary integral formulation. Siegel [17] presents a semi-analytic method to compute the interfacial dynamics in 2D Stokes flow based on conform mapping. This approach is extremely accurate but may not be valid for interface problem with arbitrary initial shape and velocity field.

Organization of this work is as follows. The governing equations are presented in Section 2. The details of the numerical methods are given in Section 3. Two examples are studied in Section 4 to demonstrate the accuracy of the numerical schemes.

2 Statement of the problem

Consider an ambient flow with velocity $\mathbf{u}^{\infty}$ past a deformable particle, as shown in Fig. 1, where Ω_1 and Ω_2 denote the regions occupied by the ambient fluid (fluid 1) and the particle (fluid 2) respectively. The governing equation is the Stokes equation combined