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Short Note

A Hybrid Numerical Method to Cure Numerical Shock Instability

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Abstract. In this note, we propose a new method to cure numerical shock instability by hybriding different numerical fluxes in the two-dimensional Euler equations. The idea of this method is to combine a "full-wave" Riemann solver and a "less-wave" Riemann solver, which uses a special modified weight based on the difference in velocity vectors. It is also found that such blending does not need to be implemented in all equations of the Euler system. We point out that the proposed method is easily extended to other "full-wave" fluxes that suffer from shock instability. Some benchmark problems are presented to validate the proposed method.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Godunov methods, numerical shock instability, carbuncle phenomenon.

1 Introduction

In the last several decades, Godunov [1] schemes based on Riemann solvers are among the most successful methods in computational fluid dynamics (CFD), which exhibit strong robustness in most situations. However, there may have some problems in extending Godunov methods to two-dimensional cases, for example, Roe solver [2] and HLLC solver [3] for the Euler equations may suffer from "carbuncle" and "odd-even decoupling" phenomena that are called numerical shock instability [4]. Some Flux-Vector-Splitting (FVS) methods such as AUSMD [5] are also found to suffer from the same problems.

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Quirk [4] suggested a framework to deal with shock instability problems by employing two different types of flux functions: one is sharp in capturing discontinuities ("full-wave" flux) and known to induce shock instability, and the other is dissipative but stable for multidimensional shocks. Quirk's approach is very useful but involves a user-defined parameter which is to determine when and where to use the Riemann solver. From then on some correction routines [6–9] have been proposed to cure the multidimensional numerical shock instability. These corrections involve the detection for grid faces deemed as susceptible to the shock instability. At grid faces, the original numerical flux functions are either modified with some special procedures resulting from multidimensional considerations, or replaced by more dissipative flux functions. These remedies have been proved to be useful, but may fail when the underlying problem involves interactions of complex flow features. Ren [10] presented a rotated Roe Riemann solver to eliminate the shock instability, where the upwind direction is determined by the velocity-difference vector. However, this method requires that the numerical flux is computed two times at each grid face, which means less efficiency in computations. Nishikawa and Kitamura [11] proposed a method which uses a weight based on the difference in velocity vector in the form of rotated fluxes. However, their method can only be applied to the Roe solver.

In this paper, we propose a new method combining the Roe solver and the HLL scheme. At first, our approach is to blend a full-wave flux "Roe" and a less-wave flux "HLL". The combined coefficients are related to velocity difference in neighbor cells and grid interface norm vector. Furthermore we find that such combination is required only for the first and the third equations in one-dimensional extended Euler system. Through the above elaborate procedure, the new method has higher resolution while keeping its robustness.

2 The hybrid method

Consider the Euler equations in two dimensions,

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}=0, \tag{2.1}$$

with

$$U=\begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix}, \qquad F(U)=\begin{bmatrix} \rho u \\ \rho u^2+p \\ \rho uv \\ u(E+p) \end{bmatrix}, \qquad G(U)=\begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2+p \\ v(E+p) \end{bmatrix},$$

where ρ is density, u and v are the velocities in x-direction and y-direction respectively, $E=\frac{1}{2}\rho(u^2+v^2)+\rho e$ is the total energy and e is the specific inner energy. Here, we only consider the ideal gas, which has a specifically equation of state: $p=(\gamma-1)\rho e$, with p the pressure and γ the specific heat ratio.