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Alternating Minimization Method for Total Variation Based Wavelet Shrinkage Model

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Abstract. In this paper, we introduce a novel hybrid variational model which generalizes the classical total variation method and the wavelet shrinkage method. An alternating minimization direction algorithm is then employed. We also prove that it converges strongly to the minimizer of the proposed hybrid model. Finally, some numerical examples illustrate clearly that the new model outperforms the standard total variation method and wavelet shrinkage method as it recovers better image details and avoids the Gibbs oscillations.

AMS subject classifications: 90C25, 52A41, 28E99, 62P35

Key words: Alternating minimization, convergence, Gibbs oscillation, wavelet shrinkage, total variation.

1 Introduction

Digital image denoising plays an important role in numerous areas of applied sciences such as medical and astronomical imaging, film restoration, and image/video coding. Throughout this paper, we suppose that Ω is an open bounded set of $\mathbb{R}^2$ with Lipschitz boundary and all the images are regarded as elements in a classical space

$$\mathcal{H} := L^2(\Omega),$$

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a separable infinite-dimensional real Hilbert space with usual inner product $\langle\cdot,\cdot\rangle$, norm $\|\cdot\|_2$. Note that as every element in $L^2(\Omega)$ can be regarded as a continuous linear functional which maps every test function to their inner product, here we consider $\mathcal{H}$ as a distributional space for convenience. Moreover, we focus on a common noisy model: an ideal image $u\in\mathcal{H}$ is observed in the presence of an additive zero-mean Gaussian noise $b\in\mathcal{H}$ of standard derivation σ. Thus the observed image $f\in\mathcal{H}$ is obtained by

$$f=u+b. \tag{1.1}$$

In the past decades, many denoising approaches have been proposed to handle this ill-posed problem. One of the widely studied techniques is the wavelet shrinkage method, which acknowledges that by applying a wavelet transform on a noisy image, random noise will contribute mainly as small coefficients in the high frequencies. Therefore, theoretically one can remove much of the noise in the image by setting these small coefficients to zero. The wavelet hard shrinkage method, which shrinks the wavelet coefficients smaller than some predefined threshold in magnitude to zero, is extremely easy and rapid to implement. Depending on the threshold, it can reduce noise rather effectively. However, it also revokes unpleasant artifacts around discontinuities as a result of Gibbs phenomenon. As artifacts in some image processing tasks may lead to great inconveniences, the wavelet hard shrinkage can not be used in these tasks without extra efforts. A development over the wavelet hard shrinkage is the wavelet soft shrinkage [15, 16], which diminishes significantly the Gibbs oscillation. Usually, the potential of wavelet shrinkage methods is rather promising when they are combined with other complex techniques which often try to take advantage of geometric information by applying wavelet-like bases better characterizing discontinuities, such as curvelets [5, 17] which can be regarded as one of the best methods among this direction. However, none of them can entirely efface the Gibbs oscillation.

Another important approach adopts regularization techniques and variational principles. Usually this approach is to determine the denoised image by minimizing a cost function consisting a data-fitting term and a regularization term

$$\min_{w\in\mathcal{H}}\frac{1}{2}\|f-w\|_2^2+\beta\mathcal{R}(w), \tag{1.2}$$

where $\mathcal{R}$ is the regularization functional and β is a positive parameter. Various possibilities for $\mathcal{R}(w)$ have been proposed in literature and earlier efforts concentrated on least squares based functionals such as $\|\Delta w\|_2^2$, $\|\nabla w\|_2^2$ and others. Though noise can be adequately reduced, these regularization functionals also impose penalty on discontinuity, conducting to rather smooth restoration images, with subtle details disappeared.

A better choice for $\mathcal{R}(w)$ was developed in [24], in which $\mathcal{R}(w)$ is the total variation of $w\in\mathcal{H}$ commonly defined by

$$\mathcal{R}(w)=TV(w):=\int_\Omega|Dw|, \tag{1.3}$$