Commun. Comput. Phys.
doi: 10.4208/cicp.250909.230410a

Vol. 9, No. 1, pp. 113-128
January 2011

Dirichlet-to-Neumann Map Method with Boundary Cells for Photonic Crystals Devices

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Received 25 September 2009; Accepted (in revised version) 23 April 2010

Available online 5 August 2010

Abstract. In a two-dimensional (2D) photonic crystal (PhC) composed of circular cylinders (dielectric rods or air holes) on a square or triangular lattice, various PhC devices can be created by removing or modifying some cylinders. Most existing numerical methods for PhC devices give rise to large sparse or smaller but dense linear systems, all of which are expensive to solve if the device is large. In a previous work [Z. Hu et al., Optics Express, 16 (2008), 17383-17399], an efficient Dirichlet-to-Neumann (DtN) map method was developed for general 2D PhC devices with an infinite background PhC to take full advantage of the underlying lattice structure. The DtN map of a unit cell is an operator that maps the wave field to its normal derivative on the cell boundary and it allows one to avoid computing the wave field in the interior of the unit cell. In this paper, we extend the DtN map method to PhC devices with a finite background PhC. Since there is no bandgap effect to confine the light in a finite PhC, a different technique for truncating the domain is needed. We enclose the finite structure with a layer of empty boundary and corner unit cells, and approximate the DtN maps of these cells based on expanding the scattered wave in outgoing plane waves. Our method gives rise to a relatively small and sparse linear systems that are particularly easy to solve.

AMS subject classifications: 78M25, 78M16, 78A45

Key words: Photonic crystal, Dirichlet-to-Neumann map, numerical simulation.

1 Introduction

Due to its periodic variation of the refractive index, a photonic crystal (PhC) [1] exhibits unusual dispersion properties and frequency intervals (i.e., bandgaps), in which propagating Bloch waves do not exist. Using the bandgap effect, waveguides and microcavities

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can be created by introducing line or point defects. In a PhC waveguide, high transmission through a sharp bend is possible [2]. Microcavities in PhCs can have very high quality factors and small mode volumes. When waveguides and cavities are combined, many PhC devices can be developed. Some examples are frequency filters [3], channel drop filters [4], waveguide branches [5], waveguide couplers [3], Mach-Zehnder interferometers [6], etc. The unusual dispersion properties and nonlinear optical effects can be used to further develop useful PhC devices.

To analyze, design and optimize PhC devices, numerical simulations are essential. Unlike the eigenvalue problem for the band structure of a perfectly periodic and infinite PhC, mathematical problems associated with a PhC device are boundary value problems in the frequency domain or initial and boundary value problems in the time domain, and they must be solved in a much larger computation domain (compared with the unit cell) using proper boundary conditions. Although different time domain methods exist [3], many authors have used the finite difference time domain (FDTD) method [7] for simulating PhC devices. For problems such as the propagation of a pulse in the device, a time domain approach is essential. However, FDTD is often used for other problems, such as the transmission and reflection spectra of a PhC device, which are more naturally formulated in the frequency domain. One reason is that FDTD is easy to understand and widely available in existing software packages. Another reason is that standard frequency domain methods, such as the finite element [8] and finite difference (in frequency domain) methods, often give rise to large, indefinite and complex linear systems that are expensive to solve. However, FDTD often requires prohibitive computer resources and produces solutions of limited accuracy.

In the frequency domain, special numerical methods can be developed to take advantage of the geometric features of the PhC devices. For ideal two-dimensional (2D) PhC devices, we can identify three geometric features: the refractive index function is piecewise constant, often with only two different values; the PhC and the defect structures are often composed of circular cylinders surrounded by a homogeneous medium, where the cylinders are either air-holes or dielectric rods; the cylinders, include the defects, often form a square or triangular lattice. A number of existing numerical methods can take advantage at least some of these geometric features. The boundary integral equation (BIE) method [9] can take advantage of first feature. We can formulate integral equations for functions defined on the dielectric interfaces. The multipole method [10–13] can take advantage of the first and second features. Around a circular cylinder, we can write down the solution in cylindrical wave expansions and solve for the coefficients. The Dirichlet-to-Neumann (DtN) map method, first developed in [14] and [15], can take advantage of all three features.

The DtN map of a unit cell Ω is the operator that maps the wave field to its normal derivative on the boundary of Ω, and it can be approximated by a small matrix. Using the DtN maps of the unit cells, we can reduce various mathematical problems for PhCs to smaller problems on the edges of the unit cells, avoiding the interiors of the unit cells completely. In earlier works, the DtN map technique has been applied to eigenvalue