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A High Order Sharp-Interface Method with Local Time Stepping for Compressible Multiphase Flows

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Abstract. In this paper, a new sharp-interface approach to simulate compressible multiphase flows is proposed. The new scheme consists of a high order WENO finite volume scheme for solving the Euler equations coupled with a high order path-conservative discontinuous Galerkin finite element scheme to evolve an indicator function that tracks the material interface. At the interface our method applies ghost cells to compute the numerical flux, as the ghost fluid method. However, unlike the original ghost fluid scheme of Fedkiw et al. [15], the state of the ghost fluid is derived from an approximate-state Riemann solver, similar to the approach proposed in [25], but based on a much simpler formulation. Our formulation leads only to one single *scalar* nonlinear algebraic equation that has to be solved at the interface, instead of the *system* used in [25]. Away from the interface, we use the new general Osher-type flux recently proposed by Dumbser and Toro [13], which is a simple but *complete* Riemann solver, applicable to general hyperbolic conservation laws. The time integration is performed using a fully-discrete one-step scheme, based on the approaches recently proposed in [5, 7]. This allows us to evolve the system also with time-accurate local time stepping. Due to the sub-cell resolution and the subsequent more restrictive time-step constraint of the DG scheme, a local evolution for the indicator function is applied, which is matched with the finite volume scheme for the solution of the Euler equations that runs with a larger time step. The use of a locally optimal time step avoids the introduction of excessive numerical diffusion in the finite volume scheme. Two different fluids have been used, namely an ideal gas and a weakly compressible fluid modeled by the Tait equation. Several tests have been computed to assess the accuracy and the performance of the new high order scheme. A verification of our algorithm has been carefully carried out using exact solutions as well as a comparison with other numerical reference solutions. The material interface is resolved sharply and accurately without spurious oscillations in the pressure field.

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1 Introduction

The simulation of compressible multiphase problems, such as reactive flows in combustion processes or flows involving phase changes, introduces many numerical difficulties associated with the treatment of the material interface. Numerical inaccuracies and spurious oscillations can occur at contact discontinuities even if diffusion and chemical reactions are not considered. In particular, unphysical oscillations in the pressure field arise at the material interface due to the change of the equation of state and the large density gradients across the interface, in particular in the case of simulations of fluids involving liquids and gases.

Different approaches have been proposed in literature. For mixtures of perfect gases with a non-constant ratio of specific heats, Abgrall [1] developed a successful quasi-conservative approach. However, the approach is limited to mixtures of ideal gases so far. A simple correction of the total energy has been proposed by Jenny et al. [22] computing the in-and outgoing volume fluxes over each cell interface such that a constant pressure in all fluids is achieved by decoding the conservative variables from the numerical fluxes. Although the numerical approaches just mentioned are successful for specific test cases, it is hard to extend them to more complex applications that contain gases and liquids with other *general* equations of state.

A completely different treatment of the material interface consists of tracking it explicitly with a level set function [27, 33], which is a signed distance function that takes the value zero at the interface. Fedkiw et al. [14,15] proposed the well-known ghost fluid method that reduces the problem of interactions of two different fluids at the material interface to two separate single-phase Riemann problems. The missing values for density, velocity and pressure on the other side of the material interface are assigned to a *ghost fluid* by a suitable procedure based on one sided extrapolation of the entropy, described in detail in [15]. This method simulates properly and efficiently general compressible multiphase flow problems avoiding spurious oscillations in the pressure and density fields at the material interface. However, Liu et al. showed from a complete analysis in [25] that the original ghost fluid method does not work correctly when applied to a strong shock impacting on a material interface, because the locations of the shock front and interface are computed in a wrong manner. Liu et al. [25] provided an improved ghost fluid method, in which a nonlinear algebraic system at the interface is solved at the interface position. It is derived from a two-shock approximation to the Riemann problem at the interface and is written for the general Mie-Grüneisen family of equations of state. The