

## The Dynamics of Lump, Lumpoff and Rogue Wave Solutions of (2+1)-Dimensional Hirota-Satsuma-Ito Equations

Ling-Di Zhang, Shou-Fu Tian\*, Wei-Qi Peng, Tian-Tian Zhang\* and Xing-Jie Yan

*School of Mathematics, China University of Mining and Technology, Xuzhou 221116, People's Republic of China.*

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**Abstract.** The Hirota-Satsuma-Ito equation in (2+1)-dimensions is studied and a new general representation of lump solutions is derived. If the lump soliton is generated by an exponentially localised line soliton, we obtain a lumpoff solution. On the other hand, if the lump soliton is generated by an exponentially localised twin plane soliton, we obtain a rogue solution. The appearance time and location of extreme rogue waves can be studied and predicted. Graphical examples demonstrate the dynamical behaviour of lumpoff and rogue waves.

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**Key words:** Lump solution, lumpoff solution, rogue wave solution, Hirota bilinear form.

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### 1. Introduction

Rogue waves attracted substantial attention in recent years because of widespread applications in nonlinear optics [10, 38], plasma science [26, 29], atmosphere research [12, 28], Bose-Einstein condensation [2] and financial problems [51]. Observations of rogue waves revealed interesting phenomena from fundamental and applied points of view [5, 7, 31, 43, 45, 46, 49]. There are a large variety of effective methods developed for solving nonlinear evolution equations — e.g. the inverse scattering transformation [1], the Lie group method [3, 27], the Darboux transformation [25], the Bäcklund transformation [37] and Hirota bilinear methods [9]. In soliton theory, special attention is paid to rational solutions analytical in all space directions, called lump waves. The set of equations having lump solutions includes (2+1)-dimensional Kadomtsev-Petviashvili equation [16], Hirota bilinear equation [15], (2+1)-dimensional Ito equation [50], (3+1)-dimensional potential

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\*Corresponding author. *Email address:* ldzhang@cumt.edu.cn (L.D. Zhang), sftian@cumt.edu.cn, shoufu2006@126.com (S.F. Tian), ts17080018a3@cumt.edu.cn (W.Q. Peng), ttzhang@cumt.edu.cn (T.T. Zhang), yanxj04@cumt.edu.cn (X.J. Yan)

Yu-Toda-Sasa-Fukuyama equation [4], (2+1)-dimensional generalised Hirota-Satsuma-Ito equation [21], etc. Various studies show the existence of lumps and their interaction solutions of integrable equations [18, 19]. Considering quadratic functions as solutions of bilinear equations leads to a new approach to the lump solutions of integrable equations [11] such as (3+1)-dimensional nonlinear evolution equation [52], (2+1)-dimensional *B*-type Kadomtsev-Petviashvili equation [30], (3+1)-dimensional *B*-type Kadomtsev-Petviashvili equation [24] and a number of other equations [6, 17, 22, 23, 32–35, 47, 48].

In this work, we consider the (2+1)-dimensional Hirota-Satsuma-Ito (HSI) equations [8, 53], viz.

$$\begin{aligned}w_t &= u_{xxt} + 3uu_t - 3u_x v_t + \sigma u_x, \\w_x &= -u_y, \quad v_x = -u,\end{aligned}\tag{1.1}$$

where  $u, w, v$  are analytic functions with respect to  $x, y, t$ , and  $\sigma$  is a real nonzero constant. Localised wave interaction,  $N$ -soliton and mixed lump line interaction solutions of the Eqs. (1.1) have been proposed in [13]. Ma [20] established interaction solutions of (1.1), whereas Liu [14] discussed its general high-order breathers, lumps and semi-rational solutions. The aim of this work is to use the symbol calculation methods [36, 39–42, 44] in order to study lump, lumpoff and rogue wave solutions of (1.1).

The paper is organised as follows. In Section 2, the Hirota bilinear form is employed to determine general lump solutions of the equation (1.1). In Section 3, we derive a lumpoff solution if the lump soliton is cut by one exponentially localised line soliton. Section 4 deals with lump solitons cut by exponentially localised twin plane solitons. In this case, we derive rogue solutions and determine when and where the rogue waves appear. A short conclusion is given in Section 5.

## 2. Hirota Bilinear Form and Lump Solutions

We now consider a more general equation obtained from the Eq. (1.1) by the transformation

$$u = 2(\ln f)_{xx}, \quad w = -2(\ln f)_{xy}, \quad v = -2(\ln f)_x,\tag{2.1}$$

where  $f$  is a real function of variables  $x, y, t$ . Substituting (2.1) into (1.1), integrating in  $x$  and using the theory of Bell polynomials, the Hirota bilinear form can be written as

$$(D_x^3 D_t + D_t D_y + \sigma D_x^2)(f \cdot f) = 0\tag{2.2}$$

with the  $D$ -operator defined by

$$\begin{aligned}&D_x^l D_y^n D_t^m (F \cdot G) \\&= \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^l \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial y'} \right)^n \left( \frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^m (F(x, y, t) \cdot G(x', y', t')) \Big|_{x=x', y=y', t=t'}\end{aligned}$$

where  $l, m, n$  are nonnegative integers. The other form of (2.2) is

$$2f_{xxxt}f - 2f_{xxx}f_t - 6f_{xxt}f_x + 6f_{xx}f_{xt} + 2f_{yt}f - 2f_y f_t + 2\sigma f_{xx}f - 2\sigma f_x^2 = 0,\tag{2.3}$$