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SOR-Like Iteration Methods for Second-Order Cone Linear Complementarity Problems

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Abstract. SOR-like modulus-based matrix splitting iteration methods for second-order cone linear complementarity problems using Jordan algebras are developed. The convergence of the methods is established and a strategy for the choice of the method parameters is discussed. Numerical experiments show the efficiency and effectiveness of SOR-like modulus-based matrix splitting iteration methods for solving SOCLCP$(A, \mathscr{K}, q)$.

AMS subject classifications: 90C33, 65H10

Key words: Linear complementarity problem, second-order cone, Jordan algebra, SOR.

1. Introduction

The l-dimensional second-order cone (SOC), also known as the Lorentz cone, is defined by

$$\mathscr{K}^l = \left\{(x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{l-1} : x_1 \geq \|x_2\|\right\},$$

where $\|\cdot\|$ is the Euclidean norm. We remark that SOC is a symmetric cone.

Let $\mathscr{K}$ be the Cartesian product of second-order cones — i.e.

$$\mathscr{K} = \mathscr{K}^{n_1} \times \mathscr{K}^{n_2} \times \cdots \times \mathscr{K}^{n_r}$$

with the positive integers $r, n_1, n_2, \ldots, n_r$ such that $n = n_1 + n_2 + \cdots + n_r$. If $n_i = 1$ for all i, $\mathscr{K}$ reduces to $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x \geq 0\}$.

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In this work, we focus on the following linear complementarity problem over SOC: Find $z \in \mathscr{K}$ such that

$$w = Az + q \in \mathscr{K} \quad \text{and} \quad \langle z, w \rangle = 0, \tag{1.1}$$

where $\langle \cdot, \cdot \rangle$ is the Euclidean inner product, $A \in \mathbb{R}^{n\times n}$ a large sparse matrix, and $q \in \mathbb{R}^n$ a given vector. In what follows, such a problem is abbreviated as SOCLCP and in the specific case (1.1) as SOCLCP$(A, \mathscr{K}, q)$. Thus SOCLCP is an extended form of the LCP.

To study SOCLCP$(A, \mathscr{K}, q)$, Fukushima *et al.* [9, 10] employed Jordan algebra tools. However, the Jordan product is not associative in general, so that the corresponding analysis differs from the classical complementarity problems approaches. Gowda and Sznajder [11, 12] studied P-and GUS-properties of linear transformations on Euclidean Jordan algebras and characterised GUS-properties of linear transformations that leave symmetric cones invariant. Recently, Yang and Yuan [28] derived verifiable sufficient and necessary conditions for the GUS-properties of SOCLCP linear transformations. Besides, the SOCLCP$(A, \mathscr{K}, q)$ has been investigated by interior-point methods [1, 29], reformulation methods with merit functions [4–6], smoothing Newton and regularisation methods [7, 13, 23]. These methods require solving a nontrivial system of linear equations at each iteration.

The sparsity of the system matrix A allows to use a matrix splitting in construction of feasible and efficient iteration methods. Murty [21] reformulated the LCP$(A, \mathbb{R}^n_+, q)$ as an implicit fixed-point equation and developed a modulus iteration method, which does not employ projections appearing in projected and general fixed-point iterations. Following this method, Bai [2] proposed a class of modulus-based matrix splitting iteration methods for large sparse LCP$(A, \mathbb{R}^n_+, q)$. Further development of such methods have been carried out in [14, 16–18, 22, 25–27]. Ke *et al.* [15] reformulated SOCLCP$(A, \mathscr{K}, q)$ as an implicit fixed-point equation based on Jordan algebra associated with the second order cone, constructed modulus-based matrix splitting iteration methods and proved their convergence. This approach extends modulus methods for LCP$(A, \mathbb{R}^n_+, q)$ to the corresponding splitting methods for SOCLCP$(A, \mathscr{K}, q)$.

In this work, we study SOR-like modulus-based matrix splitting iteration methods for SOCLCP$(A, \mathscr{K}, q)$, which are based on a Jordan algebra associated with SOC. These methods inherit good properties of the SOR method and the modulus-based matrix splitting iteration methods. We prove the convergence of the methods for SOCLCP$(A, \mathscr{K}, q)$ with the GUS-property. Besides, we propose a strategy for the parameter choice in the methods under consideration. A number of numerical tests demonstrate the efficiency and effectiveness of SOR-like modulus-based matrix splitting iteration methods.

This paper is structured as follows. In Section 2 we introduce notation and recall properties of the Jordan algebra associated with SOC. Section 3 is devoted to the SOR-like modulus-based matrix splitting iteration methods. The convergence of the methods and proper choice of the parameters involved are considered in Section 4. Section 5 contains the results of numerical experiments. Finally, our concluding remarks are given in Section 6.