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A New Compact Scheme in Exponential Form for Two-Dimensional Time-Dependent Burgers' and Navier-Stokes Equations

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Abstract. A new compact implicit exponential scheme for Burgers' and Navier-Stokes equation is developed. The method has fourth order accuracy in space and second order accuracy in time. It uses only two time levels for computation and requires nine grid points at each time level. The stability of the method is proven for linearised Burgers' equation. It is applied to a modified Taylor vortex problem. Numerical examples confirm the theoretical results and show the accuracy of the method.

AMS subject classifications: 65M06, 65M10, 65Z05, 65Y99

Key words: Compact scheme in exponential form, two-level implicit scheme, Burgers' equation, Navier-Stokes equations of motion, Taylor-vortex problem.

1. Introduction

Burgers' and Navier-Stokes equations are major objects of interest in computational fluid dynamics (CFD). During the last five decades various numerical methods have been developed for simulating viscous incompressible flows governed by these nonlinear equations. Finite difference methods (FDM) turned out to be very popular, since they are easily implemented in various situations. In the case of Navier-Stokes equations, the majority of finite difference methods have the second order accuracy that is sufficient for most of CFD problems. Among the most popular are conventional second order central and upwind schemes. For problems with smooth well-behaved solutions, these methods deliver good

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results on uniform grids. On the other hand, in convection dominated problems, they behave poorly if the mesh is not sufficiently refined. In addition, non-compact higher order discretisations require 5-grid points in both x- and y-directions that cannot be guarantied near boundary nodes. Therefore, it is important to have higher order compact schemes using 3-grid points adjacent to the boundary. Higher order compact methods are more efficient and provide more accurate numerical results.

For the time-dependent Navier-Stokes equations of motion a number of explicit and implicit methods are developed by Hirsh [9], Rai and Moin [25] and Lele [14]. These methods have fourth order accuracy in the spatial direction and second order accuracy in the time direction. For 2D advection dominated flows, Balzano [3] discussed an explicit compact method with second order time accuracy. Although explicit methods are easily implementable, they have a conditional stability limit in time step. Implicit schemes are unconditionally stable but they require matrix inversion at each advanced time level. For 1D and 2D time-dependent parabolic problems, several higher order implicit schemes are studied by Mohanty *et al.* [18,21,22], Strickwerda [30], Yanwen *et al.* [33] and Shah *et al.* [26]. Using stream-function-vorticity or stream-function-velocity formulation, Ghia *et al.* [8], Lecointe and Piquet [13], Li *et al.* [15], Spotz and Carey [29], Spotz [28], Weinan and Liu [32], Meitz and Fasel [16], Erturk and Gokcol [6], Mohanty *et al.* [17] solved the incompressible Navier-Stokes equations.

However, in 3D case such a formulation increases the number of equations and unknowns that results in higher computational cost. Tafti [31] developed an alternate formulation for the pressure equation in Laplacian form on a collocated grid for the solution of the incompressible Navier-Stokes equations. Johnson and Liu [11] studied a method for incompressible flow based on local pressure boundary conditions. A higher order finite volume method was employed by Pereira *et al.* [23], spectral method by Peyret [24], high order explicit upwind compact scheme and UGS solution algorithm by Bai *et al.* [2] in the artificial compressibility method. Other high order finite difference methods for the solution of incompressible fluid flows are discussed in [1,4,5,7,10,27].

The aim of the present work is to solve 2D time-dependent viscous Burgers' and Navier-Stokes equations of motion with appropriate initial and Dirichlet boundary conditions by a high order compact method. We propose a new exponential implicit method for general 2D nonlinear parabolic equations in line with the 2D nonlinear schemes for elliptic equations — cf. [19,20]. The method involves only two time levels and has accuracy of order two in time and order four in space. We construct an exponentially fitted method at each time level. At advanced time levels this method uses only nine grid points of a single compact cell with minimal stencil width in the x- and y-directions. Numerical simulations verify the usefulness of the proposed scheme in terms of maximum absolute (MA) errors.

The paper is arranged as follows. Section 2 deals with the discretisation of nonlinear 2D parabolic equations. The application and two-level nonlinear implicit schemes for the Burgers' and Navier-Stokes equations are discussed in Section 3. Stability of the method is considered in Section 4. Section 5 contains the results of numerical simulations. Finally, Section 6 provides the summary of this study.