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An Explicit Second Order Scheme for Decoupled Anticipated Forward Backward Stochastic Differential Equations

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Abstract. The Feynman-Kac formula and the Lagrange interpolation method are used in the construction of an explicit second order scheme for decoupled anticipated forward backward stochastic differential equations. The stability of the scheme is rigorously proved and error estimates are established. The scheme has the second order accuracy when weak order 2.0 Taylor scheme is employed to solve stochastic differential equations. Numerical tests confirm the theoretical findings.

AMS subject classifications: 60H35, 65C20, 60H10

Key words: Anticipated forward backward stochastic differential equations, explicit scheme, error estimate, second order convergence.

1. Introduction

Let $(\Omega,\mathscr{F},\mathbb{F},P)$ be a complete filtered probability space with the filtration $\mathbb{F}=\{\mathscr{F}_t\}_{t\ge 0}$ of the m-dimensional Brownian motion $W=(W_t)_{t\ge 0}$. We consider the decoupled anticipated forward backward stochastic differential equation (AFBSDE) on $(\Omega,\mathscr{F},\mathbb{F},P)$

$$\begin{aligned}
&dX_s = b(s,X_s)ds+\sigma(s,X_s)dW_s, && s\in[0,T+K],\\
&dY_s = f(s,X_s,Y_s,Z_s,X_{s+\eta(s)},Y_{s+\delta(s)},Z_{s+\zeta(s)})ds - Z_s dW_s, && s\in[0,T),\\
&Y_t = Q_t, && t\in[T,T+K],\\
&Z_t = P_t, && t\in[T,T+K],
\end{aligned} \tag{1.1}$$

where the function

$$f:[0,T]\times\mathbb{R}^d\times\mathbb{R}^p\times\mathbb{R}^{p\times m}\times L^2\big(\mathscr{F}_r;\mathbb{R}^d\big)\times L^2\big(\mathscr{F}_{r'};\mathbb{R}^p\big)\times L^2\big(\mathscr{F}_{r''};\mathbb{R}^{p\times m}\big)\to L^2\big(\mathscr{F}_s;\mathbb{R}^p\big)$$

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with $r, r', r'' \in [s, T+K]$ is referred to as the generator of the anticipated backward stochastic differential equation (ABSDE), and $Q_t, P_t \in \mathscr{F}_t$ are terminal conditions such that

$$\mathbb{E}\left[\int_T^{T+K} |Q_t|^2 + |P_t|^2 dt\right] < +\infty,$$

$b(t,x) : [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma(t,x) : [0,T] \times \mathbb{R}^d \to \mathbb{R}^{d\times m}$ the drift and diffusion coefficients, $\eta(\cdot)$, $\delta(\cdot)$ and $\zeta(\cdot)$ continuous $\mathbb{R}^+$ valued functions defined on $[0,T]$ and satisfying the conditions:

(i) There is a constant $K \geq 0$ such that for all $s \in [0,T]$ the inequalities

$$s + \eta(s) \leq T + K, \quad s + \delta(s) \leq T + K, \quad s + \zeta(s) \leq T + K \tag{1.2}$$

hold.

(ii) There is a constant $L_0 \geq 0$ such that for all $t \in [0,T]$ and for any nonnegative integrable function g the inequality

$$\int_t^T g\big(s + h(s)\big)\, ds \leq L_0 \int_t^{T+K} g(s)\, ds,$$

where

$$h(s) = \eta(s), \quad h(s) = \delta(s) \quad \text{or} \quad h(s) = \zeta(s),$$

holds.

A triplet (X_t, Y_t, Z_t) is called an L^2-adapted solution of the Eqs. (1.1) if it is $\mathscr{F}_t$-adapted, square integrable and satisfies (1.1). It is worth noting that the generator of ABSDEs depends on the conditional expectation of the future path of the solution.

An ABSDE was first introduced as a duality equation for stochastic differential delay equations (SDDEs) by Peng and Yang in [18], where the existence and uniqueness of the solutions of ABSDE are established and a stochastic control problem is solved. In the last decade, AFBSDEs attracted substantial interest because of wide applications in stochastic optimal control problems with delay [4, 10, 14, 15], stochastic differential games with delay [17, 35] and mean-field problems [5, 8]. However, exact solutions of AFBSDEs are rarely available. Therefore, it is important to develop numerical methods for AFBSDEs. We note that there is a vast literature on numerical methods for forward backward stochastic differential equations (FBSDEs) — cf. [2,3,7,16,21,24–30,32–34] and references therein. On the other hand, numerical methods for AFBSDEs are not well developed. In this work, we focus on a numerical method for decoupled AFBSDEs with generators containing a conditional expectation. An explicit numerical scheme for solving the decoupled mean-field FBSDEs with generators containing an expectation is studied in [22]. Here we propose an explicit second order scheme for decoupled AFBSDEs, rigorously prove the stability of the method, and obtain theoretical error estimates. Note that the scheme has the second order convergence rate if the weak order-two Taylor scheme is used for solving forward SDEs. We also carry out a number of numerical tests to verify theoretical findings. The tests show that our numerical scheme is stable, effective, and can achieve the second-order accuracy.