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Symbolic Computation of Lump Solutions to a Combined Equation Involving Three Types of Nonlinear Terms

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Abstract. This paper aims to compute lump solutions to a combined fourth-order equation involving three types of nonlinear terms in (2+1)-dimensions via symbolic computations. The combined nonlinear equation contains all second-order linear terms and it possesses a Hirota bilinear form under two logarithmic transformations. Two classes of explicit lump solutions are determined, which are associated with two cases of the coefficients in the model equation. Two illustrative examples of the combined nonlinear equation are presented, along with lump solutions and their representative three-dimensional plots, contour plots and density plots.

AMS subject classifications: 35Q51, 35Q53, 37K40

Key words: Soliton equation, lump solution, Hirota derivative, symbolic computation, dispersion relation.

1. Introduction

The Hirota bilinear method [3, 14] is effective in constructing soliton solutions to integrable equations generated from zero curvature equations [1, 46]. Soliton solutions are analytic, and usually exponentially localised in space and time. Assume that a polynomial P defines a Hirota bilinear differential equation

$$P(D_x, D_y, D_t) f \cdot f = 0$$

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in (2+1)-dimensions. Here D_x, D_y and D_t are Hirota's bilinear derivatives [14]. An associated partial differential equation (PDE) with a dependent variable u is often determined by some logarithmic transformation of $u = 2(\ln f)_x$ and $u = 2(\ln f)_{xx}$. Within the Hirota bilinear formulation, the N-soliton solution — cf. [13], can be presented through

$$f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij}\right),$$

where $\sum_{\mu=0,1}$ is the sum over all possibilities for $\mu_1, \mu_2, \ldots, \mu_N$ taking either 0 or 1, and the wave variables and the phase shifts are defined by

$$\xi_i = k_i x + l_i y - \omega_i t + \xi_{i,0}, \quad 1 \le i \le N,$$

and

$$e^{a_{ij}} = -\frac{P(k_i - k_j, l_i - l_j, \omega_j - \omega_i)}{P(k_i + k_j, l_i + l_j, \omega_j + \omega_i)}, \quad 1 \le i < j \le N,$$

in which k_i, l_i and ω_i, $1 \le i \le N$ satisfy the associated dispersion relation and the phases shifts $\xi_{i,0}$, $1 \le i \le N$ are arbitrary.

Recent studies show the remarkable richness of lump solutions to integrable equations, which describe various dispersive wave phenomena. Lumps are rational solutions, which are analytic and localised in all directions in space [42, 43, 49] and they can also be derived from taking long wave limits of soliton equations [47]. The KPI equation possesses diverse lump solutions [24] and its special lump solutions are generated from soliton solutions [44]. Other integrable equations which have lump solutions include the three-dimensional three-wave resonant interaction [17], the Davey-Stewartson II equation [47], the BKP equation [10, 64], the Ishimori-I equation [16], the KPI and mKPI equation with a self-consistent source [71, 72]. Moreover, non integrable equations can have lump solutions, and such equations contain the generalised KP, BKP, KP-Boussinesq, Sawada-Kotera, Calogero-Bogoyavlenskii-Schiff and Bogoyavlensky-Konopelchenko equations in (2+1)-dimensions [4, 21, 31, 37, 39, 74]. It is worth noting that the second KPI equation exhibits a new kind of lump solutions with higher-order rational dispersion relations [41]. The starting point in constructing lump solutions is to determine positive quadratic function solutions to Hirota bilinear equations [42]. Then from positive quadratic function solutions, lump solutions to nonlinear PDEs are constructed by using the logarithmic transformations.

In this paper, we would like to discuss a combined fourth-order equation in (2+1)-dimensional dispersive waves and determine its diverse lump solutions. The Hirota bilinear form plays a crucial role in our analysis [23, 42, 43, 82]. The combined nonlinear equation includes three types of fourth-order nonlinear terms and all second-order linear terms. To conduct symbolic computation of lump solutions with Maple, we will analyze two cases of the coefficients in the model equation. Illustrative examples of the considered model equation will be made, together with specific lump solutions and their three-dimensional plots, contour plots and density plots. A few concluding remarks will be given in the final section.