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Localised Nonlinear Wave Interaction in the Generalised Kadomtsev-Petviashvili Equation

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Abstract. The Hirota bilinear scheme and τ-function formalism is used in the study of localised nonlinear wave interaction structures generated by the six-soliton solutions of the generalised Kadomtsev-Petviashvili equation. Employing different sets of parameters and the long wave limit method, we consider examples of interaction of various types of waves — viz. line solitons, breathers and lumps. The dynamics of the corresponding interaction is demonstrated graphically to visualise the type of actions. The results obtained may be helpful in understanding the wave propagation in liquids containing gas bubbles.

AMS subject classifications: 37K40

Key words: Generalised Kadomtsev-Petviashvili equation, τ-function formalism, waves interaction.

1. Introduction

The two-dimensional Kadomtsev-Petviashvili (KP) equation

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma^2 u_{yy} = 0, \tag{1.1}$$

named after the authors who established it in [5], plays an important role in studying the stability of solitary waves in weakly dispersing media. Here, $\sigma^2 = \pm 1$ and the independent variables x and y represent the longitudinal and transverse spatial coordinates, respectively. If $\sigma^2 = -1$, the equation is called the KP-I equation. It is used in modeling of shallow water waves with dominating surface tension. On the other hand, if $\sigma^2 = 1$ it describes long gravity waves, such as water waves and internal waves with small surface tension, and is called the KP-II equation. Both equations are completely integrable

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Hamiltonian systems and their exact solutions can be obtained by the inverse scattering transform [30]. Nowadays, there are numerous approaches to the study of the Eq. (1.1), e.g. the nonlocal Riemann-Hilbert problem, $\bar{\partial}$-dressing method, Darboux-Backlund transformation, algebra-geometric approach, τ-function formalism, and so on.

Although KP-I equation has a variety of nonsingular localised nonlinear waves — e.g. line solitons, breathers, lumps, for KP-II equation the only nonsingular line solitons are known. Kodama and Williams [6] provided a complete classification of the spatial patterns for the KP-II line solitons, investigated their structure and resonant interaction. On the other hand, Manakov *et al.* [24] and Satsuma and Ablowitz [28] independently gave the rigorous analysis of lump solutions for the KP-I equation. In particular, they discovered that there is no phase shift when lumps collide with each other and so is when line solitons do. Zakharov [44] studied the evolution of transverse perturbations under arbitrary scale and noted the stabilisation for large wave numbers. This effect can be used in order to examine the splitting of line solitons into periodic breathers [25]. Thus the KP-I equation supports interaction solution of line solitons and breathers of the form

$$u(x,y,t) = 2[\log \tau(x,y,t)]_{xx}, \tag{1.2}$$

where

$$\begin{aligned}
\tau(x,y,t) &= 1 + 2e^{\zeta_1}\cos(\beta_1 y + 6\alpha_1\beta_1 t) + A_{12}e^{2\zeta_1} + e^{\zeta_2} \\
&\quad \times \left[1 + 2L_1 e^{\zeta_1}\cos(\beta_1 y + 6\alpha_1\beta_1 t + \phi) + A_{12}L_1^2 e^{2\zeta_1}\right], \\
\zeta_1 &= \delta_1\left[x + \alpha_1 y - \left(\delta_1^2 - 3\alpha_1^2 + 3\beta_1^2\right)t\right], \\
\zeta_2 &= \delta_2\left[x + \alpha_2 y - \left(\delta_2^2 - 3\alpha_2^2\right)t\right], \\
A_{12} &= \frac{\beta_1^2}{\delta_1^2 - \beta_1^2}, \\
L_1 \exp(i\phi) &= -\frac{(\delta_1 - \delta_2)^2 + (\alpha_1 - \alpha_2 + i\beta_1)^2}{(\delta_1 + \delta_2)^2 + (\alpha_1 - \alpha_2 + i\beta_1)^2},
\end{aligned}$$

and $\alpha_1, \alpha_2, \delta_1, \delta_2, \beta_1$ are constants. For some set of parameters, this solution reduces to the parallel interaction solution of a one line soliton and a one y-periodic breather. Fig. 1 shows that the line soliton and breather propagates along the positive x-axis, where the breather moves faster than the line soliton. They merge at the origin and the breather overtakes the line soliton.

In fact, the KP-I equation can have nonsingular interaction solutions of line solitons and lumps and of breathers and lumps. Besides, nonsingular interaction solutions can be found within the line solitons, breathers and lumps having the form (1.2) with the function

$$\begin{aligned}
\tau(x,y,t) = a_{12} + \theta_1\theta_2 + \sum_{s=3}^{5}\left\{(a_{12} + \theta_1\theta_2 + a_{2s}\theta_1 + a_{1s}\theta_2 + a_{1s}a_{2s})e^{\zeta_s}\right\} \\
+ \sum_{s=4}^{5}\left\{a_{3s}\left[\theta_1\theta_2 + (a_{23} + a_{2s})\theta_1 + (a_{13} + a_{1s})\theta_2 + a_{12}\right.\right.
\end{aligned}$$