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A Lattice Boltzmann Model for (2+1)-Dimensional Solitary and Periodic Waves of the Calogero-Bogoyavlenskii-Schiff Equation

Huimin Wang*

College of Applied Mathematics, Jilin University of Finance and Economics, Changchun 130117, China.

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Abstract. A lattice Boltzmann model is constructed to simulate the solitary and periodic wave solutions of the Calogero-Bogoyavlenskii-Schiff equation. Numerical simulations of the corresponding solitary and periodic waves show the efficiency of the method and a good computational accuracy.

AMS subject classifications: 65M10, 78A48

Key words: Lattice Boltzmann method, solitary wave, periodic wave, Calogero-Bogoyavlenskii-Schiff equation, numerical simulation.

1. Introduction

Nonlinear partial differential equations are used to describe natural phenomena and have important applications in various areas. Since the solutions of nonlinear partial differential equations can help with a better understanding of the problems considered, it is essential to find the corresponding solutions, either analytically or numerically. However, although nowadays there are many methods to determine explicit analytic solutions, the complexity of nonlinear equations makes the finding of exact solutions a very challenging problem. Therefore, numerical methods provide an alternative pathway to this problem.

In this work, we carry out numerical simulations for the solitary and periodic wave solutions of the Calogero-Bogoyavlenskii-Schiff (CBS) equation

$$\alpha\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^4 u}{\partial x^3\partial y}+\beta\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y}+\delta\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial y}=0, \tag{1.1}$$

where α,β and δ are constants. The CBS equation is used to describe the interaction between a Riemann wave along the y-axis and a long wave along the x-axis, constructed by Bogoyavlenskii and Schiff [3,4]. Almost all previous studies for solving CBS equations

*Corresponding author. *Email addresses:* whm20110605@126.com,whm780921@sina.com (H.M. Wang)

deal with analytical solutions. Many methods, such as Lie transformation method, Tanh-Coth, Hirota's bilinear, improved (G'/G)-expansion and extended tanh methods, have been used for the analytical study of the CBS equation. On the other hand, numerical methods for the CBS equation have received less attention. Here, we mainly focus on the numerical study of the CBS equation but our approach can be applied to other nonlinear equations similar to the CBS one. In particular, we employ the lattice Boltzmann method (LBM) and construct a lattice Boltzmann model for the simulation of the solitary and periodic wave solutions of the Eq. (1.1).

The LBM developed in the past 30 years, has many advantages such as the simplicity of the algorithm and the flexibility in selecting the equilibrium distribution function. It is different from traditional numerical methods based on the discretisation of the macroscopic equation. The traditional numerical methods solve the macroscopic variable directly from the discrete macroscopic equation, while LBM is based on the mesoscopic kinetic equation of the distribution function. It solves the particle distribution function but not the macroscopic equation. The macroscopic variable is obtained from the moment of the equilibrium distribution function. The use of the LBM substantially improves the accuracy of the simulations. As a powerful computational tool in computational fluid dynamics [7–9, 15], LBM found numerous applications in nonlinear partial differential equations, including the wave motion equation [32, 38], the Poisson equation [5, 12, 30], Burgers equation [14, 31, 36], Schrödinger equation [16,17,19,20,26,29,39], KdV equation [23,24,33,35,37], and some other equations [10, 13, 25, 27, 28, 34, 40]. The development of LBM in the next 20 years is also predicted, which guides the direction of future progress of LBM [21]. We note that since the time derivative term is $\partial u/\partial x$ but not the time derivative of macroscopic variable u, this creates additional problems in various numerical methods. The main task of this paper is to modify the CBS equation according to the characteristics of the equation and to construct a lattice Boltzmann model, incorporating the construction of the time derivative term, which would improve computational accuracy and lead to better simulation results. Compared with the common lattice Boltzmann model for other nonlinear partial differential equation, the lattice Boltzmann model presented in this paper is a new improved lattice Boltzmann model combined with the time derivative term construction method based on the characteristics of CBS equation.

This paper is organised as follows. In Section 2 we construct a lattice Boltzmann model for the CBS equation. In Section 3, the solitary and periodic wave solutions of CBS equation are simulated by using the lattice Boltzmann model and Section 4 contains our conclusions.

2. Lattice Boltzmann Model

Here, we develop a lattice Boltzmann model for the CBS equation. First, we choose the D2b5 lattice for the discretisation of the two-dimensional space. Let $f_\alpha^\sigma(\mathbf{x},t)$ and $f_\alpha^{\sigma,eq}(\mathbf{x},t)$, respectively, denote the single-particle distribution function and the equilibrium distribution function. The standard single relaxation lattice Boltzmann equation has the form

$$f_\alpha^\sigma(\mathbf{x}+\mathbf{e}_\alpha,t+1)-f_\alpha^\sigma(\mathbf{x},t)=-\frac{1}{\tau}\left[f_\alpha^\sigma(\mathbf{x},t)-f_\alpha^{\sigma,eq}(\mathbf{x},t)\right]+\Omega_\alpha^\sigma(\mathbf{x},t),$$