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Study on Lump Behavior for a New (3+1)-Dimensional Generalised Kadomtsev-Petviashvili Equation

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Abstract. In this paper, we investigate two dimensionally reduced cases of a new (3+1)-dimensional generalised Kadomtsev-Petviashvili equation. With symbolic computation, lump solutions are derived via searching for positive quadratic function solutions to the associated bilinear equations. Localised characteristics and lump motion are analysed and illustrated as well.

AMS subject classifications: 35A25, 35G50, 35Q35, 37K10

Key words: Bilinear method, lump dynamics, dimensionally reduced equation, symbolic computation.

1. Introduction

In the nonlinear science, more and more attention has been paid to the two-dimensional or three-dimensional nonlinear models [1, 2, 7, 10, 11, 13, 15, 17–21, 23, 24, 26, 28]. In contrast with the (1+1)-dimensional equations (one for space and the other one for time), multi-dimensional ones are more realistic in describing the nonlinear phenomena in science and engineering [4–6, 9, 27, 29]. For example, the Kadomtsev-Petviashvili (KP) equation, modeling water waves of long wavelengths with weakly non-linear restoring forces and

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frequency dispersion [10], is usually written as

$$(u_t + 6uu_x + u_{xxx})_x + \sigma u_{yy} = 0, \quad \sigma = \pm 1,$$

which is classified as the KPI equation when $\sigma = 1$ and the KPII equation when $\sigma = -1$. The KP equation is completely integrable, and its soliton solutions and lump solutions have been solved [1, 18].

The KP equation is a two-dimensional generalisation of the Korteweg-de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0,$$

where the spatial variable is generalised into two dimensions with x and y. Actually, more and more generalised KdV or KP equations are proposed, which maybe integrable or non-integrable — cf. Refs. [11, 17, 19–21, 24] and references therein.

The KP-like equation has attracted more attention recently. The generalised perturbation Darboux transformations have been reported for the (2 + 1)-dimensional KP equation and its extension by using the Taylor expansion of the Darboux matrix [25]. Fission and fusion interaction phenomena of mixed lump kink solutions for a generalised (3 + 1)-dimensional B-type KP equation has been studied by using the Hirota bilinear method [12].

In recent, a new (3+1)-dimensional generalised KP equation [24] has been introduced as

$$u_{xxxy} + 3(u_x u_y)_x + u_{tx} + u_{ty} + u_{tz} - u_{zz} = 0. \tag{1.1}$$

Via the simplified Hirota bilinear method, multiple soliton solutions to the Eq. (1.1) have been derived with the coefficients of the spatial variables left free, and the phase shifts depending on all these coefficients. It has also been proved that the Eq. (1.1) fails to pass the Painlevé integrability test although it enjoys multiple soliton solutions. Moreover, the resonant multiple wave solutions to the Eq. (1.1) have been constructed by using linear superposition principle [11].

As well known, soliton solutions are exponentially localised in certain directions, while lump solutions are a kind of rational function solutions, localised in all directions in the space [3, 14, 16]. Based on bilinear forms, one can derive both soliton solutions and lump solutions [1, 18]. The dynamics of lump, lumpoff and rogue wave solutions of (2 + 1)-dimensional Hirota-Satsuma-Ito equations has been studied through bilinear method [30]. For a fourth-order nonlinear generalised Boussinesq water wave equation, symmetry reductions and twelve families of soliton wave solutions have been derived by employing Lie symmetry method [22]. The Riemann-Hilbert approach has also been used to solve N-soliton solutions of a four-component nonlinear Schrödinger equation associated with a 5×5 Lax pair [31].

In this paper, we will focus on the dimensionally reduced cases of Eq. (1.1) and present two classes of lump solutions with symbolic computation. It is clear that the Eq. (1.1) is a (3 + 1)-dimensional model with the spatial variables (x, y, z) and the time variable t. Through a dependent variable transformation

$$u = 2\big[\ln f(x, y, z, t)\big]_x = 2\frac{f_x(x, y, z, t)}{f(x, y, z, t)}, \tag{1.2}$$