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Lump and Interaction Solutions of the (2+1)-Dimensional bSK Equation

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Abstract. In this paper, we study lump and interaction solutions of the (2+1)-dimensional bidirectional Sawada-Kotera equation. By using the Hirota bilinear method and considering a combined function with a positive quadratic function, we determine a few lump and interaction solutions of the bidirectional Sawada-Kotera equation. In order to ensure solvability of the corresponding solutions, five cases of necessary conditions for the parameters are explicitly presented. Dynamical properties of the resulting solutions are demonstrated, upon selecting appropriate values for the parameters.

AMS subject classifications: 35Q51, 35Q53, 35C99, 68W30, 74J35

Key words: Soliton, Hirota bilinear method, bidirectional Sawada-Kotera equation, lump solution, interaction solution.

1. Introduction

It is well known that the study on exact solutions of nonlinear equations plays a key role in nonlinear science. With the continuous advancement of research, solitons and important problems closely related to soliton theory have been found in many fields, such as physics, biology, medicine, oceanography, economics, and population problems. It is

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one of the most basic and important tasks in mathematics to find exact solutions of nonlinear equations. In recent years, more and more scholars have paid attention to exact solutions, particularly to nonlinear differential equations. Indeed, many powerful methods have been developed, which include the inverse scattering approach [1], the Bäcklund transformation [29, 31], Darboux transformation [28], C-K direct method [3, 11], Hirota bilinear approach [7], the auxiliary equation method [15], and the long wave limit method. In the recent study of nonlinear problems, the Hirota bilinear method has become a popular and efficient method to generate lump solutions. Very recently, in soliton theory, lump solutions have been attracting more and more attention [13, 14]. Lump solutions are a kind of regular and rationally function solutions, localised in all directions in the space [19], which appear in many physical phenomena. Amazingly, lump and interaction solutions have been presented and analysed for many nonlinear equations of mathematical physics [2,6,8,9,12,16–18,20–23,26,27,30,32,34,36–40], even for linear wave equations — cf. [24], and with higher-order dispersion relations [25].

In this paper, we will study lump and interaction solutions and their dynamics for a (2+1)-dimensional bidirectional Sawada-Kotera (bSK) equation. The (2+1)-dimensional bSK equation reads [10]

$$\begin{aligned} 9u_y - \left(5u_{xt} + 15u^3 + 15uu_{xx} + u_{xxxx}\right)_x - 15uu_t \\ + 5(\partial x)^{-1}u_{tt} - 15u_x(\partial x)^{-1}u_t = 0, \end{aligned} \tag{1.1}$$

which is subordinate to the Kadomtsev-Petviashvili equation. Here, $(\partial x)^{-1} = (\partial/\partial x)^{-1}$ and u is a function of variables x, y and t. It was formulated as a bidirectional generalisation of the Sawada-Kotera (SK) equation [33]. Its bidirectionality and relationship with the SK equation was pointed out in [4]. In view of its connection with the SK equation, the bSK equation also belongs to the Kadomtsev-Petviashvili hierarchy of B-type (BKP hierarchy) [5], the first equation of which has been shown to possess lump solutions [35]. Therefore, it will be interesting to explore lump and interaction solutions to the (2+1)-dimensional bSK equation.

The structure of the rest of the paper is as follows. In Section 2, we present a Hirota bilinear form of the (2+1)-dimensional bSK equation by the Hirota bilinear method. Then, through analysis and symbolic computations with Maple, we compute exact solutions of the bSK equation. Some figures of the resulting lump and interaction solutions with appropriate choices of the involved parameters are made to exhibit energy distributions and dynamical properties. In Section 3, we give a brief conclusion to summarize our work.

2. Lump and Interaction Solutions for the (2+1)-Dimensional bSK Equation

2.1. Bilinear form for the bSK equation

Under the bilinear transformation

$$u = 2\partial_x^2 \ln f, \tag{2.1}$$