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Local and Parallel Finite Element Schemes for the Elastic Transmission Eigenvalue Problem

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Abstract. Local and parallel finite element computations for the elastic transmission eigenvalue problem are studied in this paper. After deriving the local a priori error estimates for finite element approximations of the problem, we establish local and parallel finite element schemes, which include classical conforming finite elements and spectral elements for the problem with local low smooth eigenfunctions. The errors of the method are analysed and numerical examples show the efficiency of the schemes.

AMS subject classifications: 65N25, 65N30, 65N15

Key words: Elastic transmission eigenvalue problem, local a priori error estimate, local and parallel scheme, local defect correction.

1. Introduction

Transmission eigenvalue problems can be used to obtain estimates for the material properties of the scattering object [7, 8], and have theoretical importance for uniqueness and reconstruction in the inverse scattering theory [33]. Numerical methods for the acoustic transmission eigenvalues and the electromagnetic transmission eigenvalue problems have been developed by many researchers — cf. [9, 24, 28, 34, 43]. Similar to the acoustic and electromagnetic transmission eigenvalue problems, the elastic transmission eigenvalue problem (ETE) plays an important role in the qualitative reconstruction methods for inhomogeneous media. The ETE is a new nonlinear non-selfadjoint eigenvalue problem and its theory is far from complete [2]. Recently, the computation for the ETE has attracted the attention of the researchers: Xi *et al.* [39] studied an interior penalty discontinuous Galerkin method using the C^0 Lagrange elements (C^0IP method) based on a mixed formulation, Ji *et al.* [23] applied the secant iterative method to compute the real eigenvalues in which a fourth-order self-adjoint eigenvalue problem needs to be solved by the H^2-conforming finite element method at each iterative step, Yang *et al.* [44] studied the H^2-conforming

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methods and two-grid discretisations and [45] the mixed methods. The purpose of this paper is to study the local and parallel finite element method for the elastic transmission eigenvalue problem.

In 2000, Xu and Zhou [41] proposed some local and parallel techniques for finite element computation for elliptic boundary value problems, then they applied these techniques to eigenvalue problems [42]. The process of the local and parallel finite element algorithms is to solve the original eigenvalue problem on a relatively coarse grid and then make a correction on a locally fine grid. Later, this method has been applied to time-dependent convection-diffusion equations [25], eigenvalue problems [5, 14, 19], Navier-Stokes equations [15,16,20–22,26,27,29,30,48,49]. Among them, [14] developed the work in [41,42] and established three-scale finite element discretisations. In recent articles [4, 38], this approach has been successfully used to solve fourth-order eigenvalue problems.

In this paper, we first derive local a priori error estimate of finite element approximations for the ETE. Local a priori error estimate is one of the theoretical foundations for analysing not only local computational schemes but also graded meshes algorithms and adaptive local encryption algorithms. Compared with the work in [4], we conduct a more careful but not substantial difficult analysis since the eigenfunctions of the problem considered here are vectors. Second, we establish the local and parallel finite element schemes for solving the elastic transmission eigenvalue problem with local low smooth eigenfunctions. It is well known that spectral element methods have high accuracy for solving PDE [10, 31, 32]. Our scheme uses classical conforming finite elements and spectral elements. Then we deduce the error estimates for the approximations obtained by the local and parallel finite element scheme. When the mesh size of mesoscopic grid m is equal to the diameter of coarse grid H, our scheme performs like that in [41, 42]. As we know, adaptive computation is important and popular in practical finite element computation. Compared with adaptive methods, the main disadvantage of the local and parallel finite element method is that it cannot automatically find the encryption region. But in the problems where we can determine the singularity according to the geometric intuition and the characteristics of the problem, the local and parallel finite element method has an advantage. Through numerical experiments, we show the efficiency of the local and parallel schemes and compare the performance of the local and parallel schemes and adaptive algorithms. Theoretical analysis and numerical experiments show that our scheme works as expected.

In this paper, C is a positive constant that has nothing to do with the mesh size h and may take different values in different locations. We use $a \lesssim b$ to indicate $a \leq Cb$.

2. Preliminaries

We first introduce some notations. Let $\Omega \subset \mathbb{R}^2$ be a bounded polygonal domain, $\mathbf{x} = (x_1, x_2)^T \in \mathbb{R}^2$, $\mathbf{u}(\mathbf{x}) = (u_1(\mathbf{x}), u_2(\mathbf{x}))^T$ be the displacement vector of the wave field, $\partial_i = \partial/\partial x_i$, $\nabla\mathbf{u}$ be the displacement gradient tensor