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Nonconvex Constrained Minimisation for 3D Left Ventricular Shape Recovery Using 2D Echocardiography Data

Chi Young Ahn[1] and Sangwoon Yun[2,*]

[1] *National Institute for Mathematical Sciences, Daejeon 34047, Republic of Korea.*
[2] *Sungkyunkwan University, Department of Mathematics Education, Seoul 03063, Republic of Korea.*

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Abstract. A mathematical model in the form of a nonconvex constrained minimisation problem, aimed to determine the 3D position of LV contours using 2D echocardiography data for the entire cardiac cycle is proposed. It can be considered as a quadratically constrained quadratic program in terms of one of four variables with the others fixed. The model is solved by a proximal block coordinate descent method with cyclic order and the convergence of the algorithm is proved by using the Kurdyka-Lojasiewicz property. The model does not require unsuitable assumptions in practical environments and numerical experiments show its suitability in working with real echocardiography data.

AMS subject classifications: 49M27, 68U10, 90C20, 90C26, 92C50

Key words: Nonconvex constrained minimisation, quadratically constrained quadratic program, echocardiography, left ventricle, 3D reconstruction.

1. Introduction

Despite the usefulness of real-time 3D echocardiography (RT3DE) [16,17], it has its limitations such as relatively high cost and poor temporal/spatial resolutions compared to 2D echocardiography. Because of that, the 2D echocardiography is more preferable in clinical practice. Thus most of analysis and diagnostic tools are still performed with measurements in 2D slices. However, it is noteworthy that recent studies on ultrafast ultrasound imaging are expected to improve the poor resolution performance of RT3DE and to provide robust analysis of the LV behavior [3,6].

This work is the continuation of our studies [1,2] on the 3D left ventricle (LV) border recovery from 2D echocardiography data. This topic attracts substantial interest because the dynamic 3D motion of the heart is observed without using 3D imaging scanners. The main

*Corresponding author. *Email addresses:* `chiyoung@nims.re.kr` (C. Y. Ahn), `yswmathedu@skku.edu` (S. Yun)

problem consists in obtaining image data of multiple images and making an interpolation of the cardiac chambers in 3D. Thus the 3D representation of the LV shape requires the 3D position information of multiple 2D imaging planes. There are various studies on recovering 3D LV shape from 2D echocardiography — cf. [9, 11, 19], where additional devices, such as motor or position sensors are used for rotating 2D imaging probe mechanically or by sensing the probe positions. The 3D position and orientation of 2D image planes are spatially pre-determined or tracked using a sensor attached to a probe [15].

Similar to [1, 2], this paper focuses on the problem of establishing the 3D position of imaging planes associated with multiple 2D apical echocardiograms, which are obtained with involvement of any additional device. In particular, previously we considered mathematical models based on the fact that the angles between apical long-axis 4, 2 and 3-chamber views are approximately 60° toward the each other [14], cf. Fig. 1. It was also assumed that the imaging planes corresponding to the three views intersect at the same axis and the circumferential length of mitral annulus with small variation throughout the whole cardiac cycle is in the fixed range. Under such assumptions, the models were applied to still image data.

The aim of this work is to build a new 3D recovery model without the assumptions used in the previous models. Those assumptions are removed by using moving image data acquired during the whole cardiac cycle. The only assumption is that an imaging probe is fixed without any movement for any heartbeat period smaller than 1 second. This means that throughout the entire cardiac cycle, the apical long-axis 2-, 3- and 4-chamber views are fixed and the motion of LV borders is observed only on fixed planes. It uses 2D echocardiography data obtained during the entire cardiac cycle. As shown in Fig. 3, it is expected that the distance between the LV control points moving in every fixed plane holds a proper distance in space. Wrong positions of the planes may lead to a very irregular and large distance in space between the control points. Hence, we propose a model maintaining

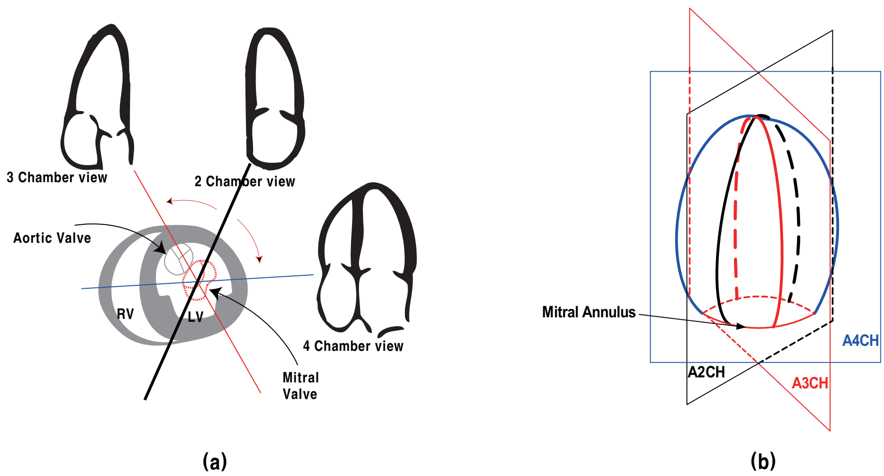

Figure 1: Apical 4-, 2- and 3-chamber views of cardiac images. The images are obtained by clockwise and counterclockwise rotating of the scanning probe by 60°.