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Novel Rogue Waves for a Mixed Coupled Nonlinear Schrödinger Equation on Darboux-Dressing Transformation

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Abstract. Focusing-defocusing mixed coupled nonlinear Schrödinger equation of localised waves in a two-mode nonlinear fiber is investigated. Novel localised wave solutions are constructed by employing the Darboux-dressing transformation. The set of such solutions includes rogue waves on the soliton background. In addition, for the main characteristics of these solutions, we give the graphs to make readers more aware of the characteristics of these solutions. Hopefully our results can be used to help enrich rogue waves phenomena in nonlinear wave field.

AMS subject classifications: 65M10, 78A48
Key words: Mixed coupled nonlinear Schrödinger equation, Darboux-dressing transformation, breather wave, rogue wave.

1. Introduction

Rogue waves are used to describe huge catastrophic waves unexpectedly arising on relatively calm ocean surface [12] and to characterise extreme wave events in optics [20], plasma [16], Bose-Einstein condensate [5], finance [29, 30], and so on. It is the common belief that rogue waves have three main characteristics:

1) The amplitude of the wave is more than twice (or larger) than the average amplitude of the significant wave height [1].

2) They appear from nowhere and disappear without trace [2].

3) The probability distribution function of the amplitude obeys the unusual L-shaped statistics, which means that the frequency of the wave is higher than predicted by the classical Gaussian distribution [1, 20].

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Recently, rogue waves have been encountered in optical fibers, deep water waves and other fields where nonlinear Schrödinger (NLS) equations are employed. In particular, Peregrine [19] constructed a first-order rational solution of the NLS equation by mathematical method, which is a formal description of a single rogue wave. This solution, the peak amplitude of which is three times of the average height, was later named after him. The rogue waves attracted a great attention in recent years — cf. [6, 7, 10, 31, 34, 35], and it is worth noting their close connection to NLS and coupled nonlinear Schrödinger equation(CNLS), Li *et al.* [13] determined reduced and non-reduced vector rogue wave solutions of CNLS using the generalised Darboux transformation (DT), Feng *et al.* [8] employed DT in order to construct multi-breather solutions of NLS on the background of elliptic functions and expressed them via theta functions, Zhang *et al.* [33] used DT in new localised wave solutions; and so on.

On the other hand, the Darboux-dressing transformation has been used in the study of the classical Schrödinger equation [17], integrable vector nonlinear Schrödinger equations [18], the Manakov system [23], the Kundu-nonlinear Schrödinger equation [25], the coupled cubic-quintic nonlinear Schrödinger equations [28] and in other fields [24,26,27,32].

Numerous works are devoted the two-component case (as so called the Manakov system)

$$
\begin{aligned}
&iu_t + \frac{1}{2}u_{xx} + \sigma\left(|u|^2 + |v|^2\right)u = 0, \\
&iv_t + \frac{1}{2}v_{xx} + \sigma\left(|u|^2 + |v|^2\right)v = 0,
\end{aligned}
\tag{1.1}
$$

where $u(x,t)$ and $v(x,t)$ are wave envelopes, and x and t are, respectively, transverse and longitudinal coordinates [3, 9]. Every subscripted variable in the Eqs. (1.1) refers to the partial differentiation. If $\sigma = 1$, the equations represent the defocusing case, and if $\sigma \neq 1$ the focusing case.

In this work, we consider breather and rogue waves of the focusing-defocusing mixed coupled nonlinear Schrödinger equation (mCNLSEs)

$$
\begin{aligned}
&iu_t + \frac{1}{2}u_{xx} + \left(|v|^2 - |u|^2\right)u = 0, \\
&iv_t + \frac{1}{2}v_{xx} + \left(|v|^2 - |u|^2\right)v = 0,
\end{aligned}
\tag{1.2}
$$

where u and v are respectively related to the focusing and defocusing type nonlinearities [21]. The terms $|u|^2u$, $|v|^2v$ and $|v|^2u$, $|u|^2v$ are self-phase and cross-phase modulations. The Eqs. (1.2) can be regarded as a mixture of the defocusing and focusing conditions of the Manakov system (1.1) and are vigorously studied. For example, Tian *et al.* [21] considered initial-boundary value problems related to the Fokas method, Vijayajayanthi *et al.* [22] studied bright-dark solitons and their collisions in mixed N-coupled nonlinear Schrödinger equations, Kanna *et al.* [11] investigated the soliton collisions with a shape change by intensity redistribution, Ling *et al.* [14] constructed vector rogue wave and bright-dark rogue wave solutions by using the Darboux transformation. However, to the best of the our knowl-