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Stable Computation of Least Squares Problems of the OGM(1,N) Model and Short-Term Traffic Flow Prediction

Qin-Qin Shen[1], Yang Cao[1,*], Bo Zeng[2] and Quan Shi[1]

[1]*School of Transportation and Civil Engineering, Nantong University,*
Nantong 226019, P.R. China.
[2]*College of Business Planning, Chongqing Technology and Business University,*
Chongqing 400067, P.R. China.

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Abstract. The optimized grey multi-variable model, used to overcome the defects of the grey multi-variable model, is studied. Although this model represents a substantial improvement of the grey multi-variable one, unstable computation of the grey coefficients arising in ill-posed problems, may essentially diminish the model accuracy. Therefore, in the case of ill-posedness we employ regularization methods and use the generalized cross validation method to determine the regularization parameters. The methods developed are applied to the urban road short-term traffic flow prediction problem. Numerical simulations show that the methods proposed are highly accurate and outperform the grey multi-variate, the autoregressive integrated moving average, and the back propagation neural network models.

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Key words: Grey multi-variable model, least squares problem, ill-posed problem, regularization technique, traffic flow prediction.

1. Introduction

As an important branch of grey system theory [14], the grey prediction model attracted considerable attention because of the high efficiency and accuracy in time series prediction problems, especially in the case of small samples and incomplete information [27, 46]. It has numerous applications in such areas as indirect measurements of tensile strength [34, 35, 48], energy consumption [17, 32], traffic and amount of motor vehicles prediction [10, 12, 19, 41, 42, 49], industrial and agriculture production prediction [28, 47], etc. It can be regarded as a special differential equation model and it is often abbreviated as GM(p,q), where p is the order of the corresponding differential equation and q the number of variables.

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[*] Corresponding author. *Email address:* `caoyangnt@ntu.edu.cn` (Y. Cao)

There is a variety of grey models [10,17,28,32,34,35,41,42,46–49], but the first order differential equations are used more often. Such models are divided into two categories — viz. single-variable grey models GM(1,1) and multi-variable grey models GM(1,N), cf. [44] for an historic review.

The original GM(1,N) model proposed by Deng [14], has a characteristic system variable and $N-1$ related factor variables. Its aim is to extend the grey model from single-variable to multi-variable situation. Since the GM(1,N) models take into account all relevant factors connected with the system change, they are more useful in simulating operation laws and the development of systems [48]. Nevertheless, the original GM(1,N) model is mainly used to analyze the closeness of the system characteristic and relevant variables and only episodic in practical problems. The reason for this is that the deriving term in the model is treated as a constant, which leads to a wrong solution — cf. [34,36]. On the other hand, there are various suggestions aimed to correct the process and to improve the prediction accuracy. They can be classified as follows:

1. Improvement of the model structure to obtain more reasonable models [28,31,36, 48,49].
2. Finding new accumulated generating operators to derive smoother sequences [32, 40,42].
3. Optimizing related parameters and background value to get more stable models [39, 50].
4. Deriving discrete solutions to get more compatible models [15,43,48,49].
5. Studying the hysteresis of independent variables in order to obtain models better adjusted to the actual situation [16].

In particular, the optimized GM(1,N) model proposed by Zeng *et al* [48] is a quality improvement of the GM(1,N) model. In theory, the OGM(1,N) overcomes three obvious defects of the traditional GM(1,N) model — viz. mechanism, parameter and structure defects and numerical examples in [48] confirm that the OGM(1,N) model has a much better prediction accuracy than the traditional GM(1,N) one.

Applying OGM(1,N), GM(1,N) and other grey models to practical problems, we have to solve in advance a linear least squares problem — viz.

$$Bp = Y, \tag{1.1}$$

in order to determine the so called grey coefficients [6–9]. Note that $B \in \mathbb{R}^{(n-1)\times(N+2)}$ is a dense matrix, $p \in \mathbb{R}^{N+2}$ an unknown grey coefficients vector, $Y \in \mathbb{R}^{n-1}$ a given vector, N the number of variables, and n the length of the variable sequence. If the coefficient matrix B of the least squares problem (1.1) is well-conditioned, we can use the standard method to obtain a least squares solution [11]. However, if the coefficient matrix is ill-conditioned in the sense that the coefficient matrix has many singular values close to the origin or the condition number of the coefficient matrix is too large, the grey coefficients