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Two-Step Modulus-Based Synchronous Multisplitting Iteration Methods for Nonlinear Complementarity Problems

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Abstract. Two-step modulus-based synchronous multisplitting and symmetric modulus-based synchronous multisplitting accelerated overrelaxation iteration methods are developed for solving large sparse nonlinear complementarity problems. The methods are based on the reformulation of the corresponding problem as a series of equivalent implicit fixed-point equations. This approach includes existing algorithms as special cases and present new models. The convergence of the methods is studied in the case of H_+ system matrices. Numerical results confirm the efficiency of the methods proposed.

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Key words: Nonlinear complementarity problem, two-step modulus-based synchronous multisplitting, iteration method, H_+-matrix, H-splitting.

1. Introduction

Let $\mathbb{R}^{m\times n}$ be the space of $m \times n$ real matrices and $\mathbb{R}^n := \mathbb{R}^{n\times 1}$. In this work, we concentrate on the following nonlinear complementarity problem NCP(q,A): Find a pair of real vectors $u, v \in \mathbb{R}^n$ such that

$$u \geq 0, \quad v := Au + q + \Phi(u) \geq 0, \quad u^T v = 0, \tag{1.1}$$

where $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ is a sparse matrix, $q = (q_1, q_2, \ldots, q_n)^T \in \mathbb{R}^n$ a real vector, $\Phi : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous diagonal mapping — i.e. each component Φ_i of Φ is a function

$$\Phi_i = \Phi_i(u_i).$$

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of the i-th variable u_i. Besides, by $\geq$ we denote the componentwise partial ordering in $\mathbb{R}^{m\times n}$, cf. Definition 2.1 below, and by A^T the transpose of $A \in \mathbb{R}^{m\times n}$.

Complementarity problems arise in various computing and engineering applications, including linear and quadratic programming, contact problems in elasticity, economies with institutional restrictions upon prices, variational inequality problems, optimal stopping in Markov chain, free boundary problem of fluid dynamics, and optimal legalization for circuit designs — cf. Refs. [12–14, 16, 17, 37]. A special case of NCPs (1.1), the linear complementarity problems (LCPs) have numerous practical applications and there are a variety of feasible and efficient iteration methods developed for their solution. In particular, we note projected relaxation methods [1, 2, 14, 20, 28, 36], general fixed-point iteration methods [33] and multisplitting iteration method [3, 5–7].

The modulus method [27, 34] based on the reformulation of LCPs as an equivalent implicit fixed-point equation, is a classic iteration method for solving such problems. Dong and Jiang [15] modified the modulus method by introducing a parameter α into the implicit fixed-point equation and determined an optimal parameter for symmetric positive-definite system matrices A. In the case of large system matrices, the corresponding linear complementarity problem can be handled by the matrix splitting technique [8]. In particular, Bai [4] introduced a series of modulus matrix splitting (MMS) iteration methods, and analysed their convergence for positive-definite and H_+ system matrices. In practical computation, these methods are more effective than the others such as the extrapolated modulus method [19]. The conditions of the convergence of the MMS methods have been improved by Zhang and Ren [46], and Xu and Liu [42] obtained better convergence results for H_+ system matrices. The study of the MMS method accelerated by matrix splitting and the relaxation strategy has been undertaken in [30, 38, 47, 48]. Li [29] extended the modulus-based matrix splitting iteration method to a general case. Multisplitting iterative methods are a powerful tool to enlarge the scale of problem and speed up the computation. Using this approach, Bai and Zhang constructed modulus-based synchronous multisplitting iteration methods (MSM) [9], modulus-based synchronous two-stage multisplitting iteration methods (MSTM) [10], and two-step modulus-based synchronous multisplitting iteration methods (TMSM) [45], suitable for implementation on multiprocessor systems.

However, for general functions $\Phi(u)$, (1.1) belongs to the set of NCPs [17, 21]. Since such problems have important applications in different fields, there are numerous iteration methods developed for their solution [26, 31, 32, 39]. For example, the MMS methods are also used to solve NCPs. In particular, Huang and Ma [25] extended MMS approach to handle a class of weakly nonlinear complementarity problems with positive definite system matrices A and Lipschitz continuous nonlinear parts. Hong and Li [22] proposed modulus-based matrix splitting iteration methods for solving implicit complementarity problems. Recently, Xia and Li [43] presented another modulus-based matrix splitting iteration method for the problem (1.1). Furthermore, Xie *et al.* [44] introduced two-step modulus-based matrix splitting iteration method, while Wu *et al.* [41] studied a modulus-based synchronous multisplitting iteration method for the problem (1.1).

The present work is devoted to new two-step modulus-based synchronous multisplitting iteration methods for the problem (1.1). We reformulate this problem as a series of fixed-