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Simulations of Cuff-Tissue-Bone-Artery Systems for Blood Pressure Measurements

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Abstract. The elastic mechanics of cuff-tissue-bone-artery systems is studied by using the phase-field method. It is shown that such factors as the Young modulus of the tissue, thickness and elasticity of the blood vessel wall, and blood and cuff pressures significantly influence the oscillation waveforms of the cuff pressure and cause difficulties in decoding the cuff measurements. In addition, our simulations point out at additional information in the oscillometric waveform, which can be of help in the blood pressure decoding. The development of the next-generation instruments for blood pressure measurements requires further studies of cuff-tissue-bone-artery systems and elastic properties of human tissue and blood vessel walls.

AMS subject classifications: 35J57, 74B20

Key words: Blood pressure measurement, cuff, artery wall, tissue, oscillometric waveform.

1. Introduction

The blood pressure plays an important role in diagnosis and prevention of diseases such as hypertension and stroke [10, 18, 19, 22]. Accurate measurements of blood pressure attract wide interests in both industry and academia. The most common techniques in blood-pressure measurements are based on auscultation and oscillometric methods [4, 6, 12, 16, 17, 24, 30, 32, 34, 41, 47], where an inflatable cuff surrounding the arm (or wrist) is used to pressurize the tissue.

Auscultation method is performed by listening to the Korotkoff sound. It provides systolic and diastolic blood pressure by the cuff pressure at the appearance and disappearance of Korotkoff sound [4, 32, 41]. Blood pressure obtained in auscultation method is regarded as the gold standard of blood pressure measurements in current medicine. Nevertheless, the origin of the Korotkoff sound remains unclear so far. Due to its technical requirements

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and labor costs, auscultation method is not a suitable solution for daily monitoring of blood pressure.

Oscillometric method firstly proposed by Marey [30] is based on the oscillometric waveform envelopes (OWE) obtained by the pressure sensors in cuffs. This method has evolved into a few different algorithms, including the maximum amplitude algorithm [12, 16, 17] and the slope method [24, 34]. Due to its simplicity, such technology has been widely used in physical examination and wearable equipment [6, 30, 47]. However, the accuracy of these algorithms is often low because of insufficient theoretical support and population differences.

Statistical studies and theoretical models have been developed to understand the oscillometric pressure waveforms measured in the cuff. It is observed that the peak of the OWE is reached when the cuff pressure is roughly equal to the mean blood pressure [5, 31, 35]. Although there is no clear mechanical explanation, it is believed that this relation between the OWE and cuff pressure comes from the compliance of arteries, which depends on the elasticity of the arterial vessel walls [5, 31, 35]. In a few later models, the OWE is reproduced by coupling the arterial wall model and the cuff model but neglecting the influence of tissue elasticity [3, 23]. In a few recent modeling and simulation studies, the influence of the tissue [25, 42, 43] and the bones [11] is taken into account. The main features of the OWE are successfully reproduced in these studies, though a nontrivial assumption of a constitutive relation is required for the blood vessel wall. The significant factors influencing the waveforms have not been fully recognized and carefully analyzed. Further modeling and simulation studies can help to find out these factors and useful information for blood-pressure decoding.

The phase-field method is a mature technique for tracking moving interfaces. The main advantages of this method are:

1. It can be naturally used to incorporate multiple phases separated by complex interfaces.
2. It can be spontaneously used to track the movement of interfaces and apply surface stresses.
3. It is friendly to structural grids.

Various applications of the method appear in the modeling and simulation of phase transition [29, 36, 45], phase separation [20, 21, 40], topology optimization [7, 8, 38, 44], solidification [2, 26, 27, 37, 39], multi-phase flows [48, 49], and viscous fingering [9, 15, 33]. Recently, the phase-field method has been used in simulation of elasticity problems with multiple phases and large deformation [46].

The aim of this work is to figure out the factors significantly influencing the OWE and to determine useful information in the oscillometric waveform measured in the cuff. A two-dimensional model of the cuff-tissue-bone-artery (CTBA) system is used in our simulations. The computational difficulties of the complex CTBA systems introduced by multiple domains (tissue, bone, and vessel wall) and complex boundary conditions (cuff pressure and