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Stability of High-Order Finite-Difference Schemes for Poroelastic Wave Simulation

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Abstract. The stability of high-order finite-difference schemes on a staggered-grid for two-dimensional poroelastic wave equations with spatially varying material parameters is studied. Using the energy method, we obtain sufficient stability conditions. This allows to find suitable time and spatial steps according to material parameters and the difference scheme coefficients. Two numerical examples verify the theoretical analysis and show that the corresponding range for the time step is close to that in the necessary condition. The perfectly matched layer is adopted in order to eliminate boundary reflections.

AMS subject classifications: 65M10, 78A48

Key words: Stability, poroelastic wave equation, energy method, high-order scheme, wave simulation.

1. Introduction

Numerical modelling of the wave propagation based on acoustic and elastic wave equations has a number of important applications, including the search for geological structures containing oil and gas reservoirs. In contrast to acoustic and elastic models, which are usually used in exploration geophysics, real media in reservoir area can be also considered as porous media saturated with oil and gas [7]. Note that the description of poroelastic media uses various parameters — e.g. elastic constants, permeability, fluid density, etc. The theory of wave propagation in poroelastic media was established by Biot [4, 5] in fiftieth of the last century and is widely used since then. The poroelastic wave equation is usually called the Biot's equation.

There are many numerical methods developed to solve poroelastic wave equations, including finite-difference methods [13, 23, 31, 35, 39, 44], finite volume methods [16, 33,

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46], finite element methods [2,8,11,33,43], the discontinuous Galerkin methods [9,10,14, 15,27,38], and a multiscale method [47]. Each of them has advantages and disadvantages. In particular, finite-difference (FD) methods are very popular due to a high computational efficiency and simplicity of computational implementation. In this paper, we focus on the stability of high-order FD schemes for poroelastic wave equations with spatially varying material parameters.

The stability conditions of FD schemes play an important role. They can suggest how to effectively choose computational parameters — e.g. time and spatial steps. For homogeneous media, there are certain known stability conditions of FD schemes for poroelastic wave equations. Thus considering the two-dimensional (2D) Biot's equation, Masson *et al.* [30] investigated the stability of a difference scheme by von Neumann method. Itzá *et al.* [25] also used von Neumann method to show the stability of optimal implicit 2D finite differences for poroelastic media equations. However, the range of stability for the optimal implicit scheme is larger than the one in [30] for an explicit scheme. In [35], the 2D explicit stability of 3D rotated and standard staggered-grid schemes with fourth-order in space and second-order in time is obtained. Alkhimenkov *et al.* [1] studied the stability of different discretization schemes with the second-order accuracy in space for the Biot's equation. On the other hand, in order to investigate the stability of high-order schemes with the fourth-order accuracy in space, the approach of [1] has to be generalized. In [42], the stability of 3D high-order staggered-grid schemes in homogeneous media has been investigated by von Neumann method. It is worth mentioning that this approach was used in order to study the stability of all schemes mentioned. However, the method is only applicable to the Biot's equation with homogeneous material parameters and only if there are plane wave solutions. For equations with spatially varying elastic coefficients, the von Neumann method does not work. Thus the stability of finite-difference schemes used in poroelastic wave simulations is still not well studied, partially because the previous investigations are limited to homogeneous media. On the other hand, here we use the energy method in order to analyze the stability of high-order staggered-grid schemes for poroelastic wave equations with spatially varying material parameters. Other applications of this method to wave equations can be found in [22, 34, 36]. The motivation of this work is to establish explicit numerical stability conditions of high-order finite-difference schemes for poroelastic wave equations. We note that the consistency of the differences is obvious. Therefore, by the Lax equivalence theorem the numerical stability can guarantee the convergence of numerical computations for the linear difference schemes [37].

The rest of this paper is organized as follows. The stability conditions of the numerical schemes under consideration are established in Section 2. Section 3 provides numerical results illustrating the theoretical analysis. Our conclusions are given in Section 4.

2. Theory

Writing down the governing equation, we construct high-order difference schemes on a staggered grid and derive their stability condition by using the energy method.