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Dynamical Behavior of a Lotka-Volterra Competitive System From River Ecology

Jinyu Wei[1,2,3] and Bin Liu[2,3,*]

[1]*School of Statistics and Mathematics, Hubei University of Economics, Wuhan 430000, Hubei, P.R. China.*
[2]*School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, Hubei, P.R. China.*
[3]*Hubei Key Laboratory of Engineering Modeling and Scientific Computing, Huazhong University of Science and Technology, Wuhan 430074, Hubei, P.R. China.*

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Abstract. This work is devoted to the study of a two-species competition model in advective homogenous environment from the river ecology. We assume that two species live in a special river where the upstream end has free-flow boundary conditions. This means that the upstream end is linked to a lake. On the other hand, at the downstream end the population may be exposed to differing magnitudes of individuals loss. We mainly study the influence of inter-specific competition intensities on the competition outcome and show that the contest is very complex — viz. either one of competitors becomes a single winner (exclusion), or both populations coexist, or both species go to extinction.

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1. Introduction

The dynamical models expressed by reaction-diffusion equations have been actively studied in recent years. In particular, such models can describe the uneven distribution of individuals across an area. This is an important issue in various fields of natural sciences and the two-species Lotka-Volterra competition-diffusion system is one of the most popular models used to investigate the problems mentioned — cf. [2, 8, 9, 22].

In addition to traditional reaction-diffusion models, there are numerous studies focused on spatial population dynamics in advective environment. The advection is an inducement of an individual movement in a given direction caused by external environmental forces

*Corresponding author. *Email addresses:* weijy@hust.edu.cn (J. Wei), binliu@mail.hust.edu.cn (B. Liu)

like lake water columns, streams, or rivers [2–5, 18–20]. The advection terms can be incorporated in these classical Lotka-Volterra competition-diffusion systems which results the competition-diffusion-advection systems. Most obviously, it occurs in rivers where individuals can be swept downstream by water flows. In order to study different ecological scenarios, Lou and Lutscher [14] considered various boundary conditions at the upstream and downstream ends. More exactly, the competition-diffusion-advection system with various boundary conditions has the form

$$\begin{aligned}
&u_t = d_1 u_{xx} - \alpha_1 u_x + u[r - u - av], && 0 < x < L, && t > 0,\\
&v_t = d_2 v_{xx} - \alpha_2 v_x + v[r - cu - v], && 0 < x < L, && t > 0,\\
&d_1 u_x(x,t) - \alpha_1 u(x,t) = b_1 \alpha_1 u(x,t), && x = 0, && t > 0,\\
&d_1 u_x(x,t) - \alpha_1 u(x,t) = -b_2 \alpha_1 u(x,t), && x = L, && t > 0,\\
&d_2 v_x(x,t) - \alpha_2 v(x,t) = b_1 \alpha_2 v(x,t), && x = 0, && t > 0,\\
&d_2 v_x(x,t) - \alpha_2 v(x,t) = -b_2 \alpha_2 v(x,t), && x = L, && t > 0,\\
&u(x,0) = u_0 \geq, \not\equiv 0, \ \ v(x,0) = v_0 \geq, \not\equiv 0, && 0 < x < L,
\end{aligned} \tag{1.1}$$

where $a > 0$, $c > 0$ are the inter-specific competition intensities, u, v the population densities of two aquatic competing species, $d_1 > 0$, $d_2 > 0$ random diffusion rates of the two species, and $\alpha_1 > 0$, $\alpha_2 > 0$ the effective advection rates caused by unidirectional water flow. Besides, r is the intrinsic growth rate or the local carrying capacity and L the size of the habitat. In what follows, $x = 0$ and $x = L$ are called the upstream and downstream ends. The parameters $b_1 \geq 0$ and $b_2 \geq 0$ are used to measure the respective loss rate of individuals at the upstream and downstream ends which are relative to the flow rate — [14]. More specifically, let us consider b_2 as an example. It is easily seen that $b_2 = 0, 1$ produces no-flux Neumann boundary conditions. The corresponding problem can be used in order to describe the scenario stream to lake [14, 27]. If $b_2 > 1$, it means that random and directed movements will cause population loss. Moreover, sufficiently large coefficient b_2 reflects a severe loss of individuals at the downstream end, which in turn indicates that the downstream area is unfavorable for organisms to survive. Formally, we can regard $b_2 = \infty$ as the Dirichlet boundary condition $u(L,t) = v(L,t) = 0$ for $t > 0$ which can be used to model the situation stream to ocean [24].

We note that various special cases and variants of the system (1.1) have been qualitatively investigated in last years. Under no-flux boundary conditions — i.e. if no individual passes through the upstream and downstream ends, Lou *et al.* [15] confirmed that a weaker advection is more beneficial for species to exclude its competitor when $d_1 = d_2$ and $\alpha_1 \neq \alpha_2$. For unequal movement rates $d_1 \neq d_2$ and $\alpha_1 \neq \alpha_2$, Zhou [31] found that the strategy of faster diffusion together with slower advection is always favorable. This result can be viewed as a generation of [15]. For other boundary conditions — e.g. if $d_1 \neq d_2$ and $\alpha_1 = \alpha_2$, Lou and Zhou [17] suggested that the competitor with faster diffusion rate will displace the slower one — i.e. the faster diffusion evolves for $b_1 = 0$ and $b_2 \in [0, 1)$. If $d_1 = d_2$ and $\alpha_1 \neq \alpha_2$, Xu *et al.* [30] considered the case $b_1 = 0$, $1/2 \leq b_2 \leq +\infty$ and showed that a weaker advection is more favorable for species to survive, thus extending results [15]. For higher spatial dimension studies, we refer the reader to [29, 32].