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Framework and Numerical Algorithm for a Phase Field Fracture Model

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Abstract. A phase field fracture model for quasi-brittle material in 2D is implemented in Abaqus software. The phase field damage variables 0 and 1 define undamaged and damaged regions of the material and simplify crack surface tracking. On the other hand, one has to use a fine spatial discretization for the smooth distribution of the phase field variable regularized by a small length scale parameter, which makes the method computationally expensive. At the fully damaged regions both the stiffness and stress reach zero. The displacements and damage are determined by a staggered approach, and a few standard benchmark fracture problems are used to demonstrate the work of the phase field fracture model under consideration.

AMS subject classifications: 65M10, 78A48

Key words: Phase field method, fracture, sharp crack, staggered approach and nonlinear finite element method.

1. Introduction

Fracture in engineering materials and structures is one of the most common modes of failure, and it is essential to check for a possible fracture and to prevent the progress of cracks in the material while designing a structure. Therefore, it is important to understand the failure behavior of various materials. Since experimental tests can be expensive and sometimes impossible to do, numerical models have gained a lot of interest in recent years. To predict the fracture failure, various numerical methods and approaches have been developed. Griffith [12] proposed a theory of brittle fracture, where the crack propagation is determined based on the energy requirements to create new crack surfaces. Nevertheless, this theory cannot predict the crack nucleation and other phenomena such as crack kinking and branching.

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Numerical fracture models can be considered in continuous and discontinuous settings by using the tools and methods of continuum damage mechanics, linear elastic fracture mechanics [12], and cohesive zone models [10]. Such approaches require additional criteria for crack initiation and propagation [6] and for crack branching [7]. Recent developments are focused on non classical theories, making use of length scale parameters to regularize solutions. They have built-in criteria to predict the onset of crack and produce mesh independent results for crack propagation. Hence, there is an increased interest and popularity for approaches such as the phase field models [9, 16] and peridynamic models [13]. The phase field models are based on the energy minimization principle and can automatically predict the crack initiation, growth and coalescence. Such models can also show the branching and merging of different cracks without using any additional criteria. The 2D model considered in this paper can be easily extended to three-dimensional ones since numerical implementation is straightforward in any dimension. Along with these advantages, the method has a few drawbacks. In particular, it requires a fine mesh to accurately solve the gradient terms present in the model, so that the computational cost is high. Besides, in the case of dynamic loading, the problem of the crack tip leads to inaccurate prediction of the crack velocity.

The first phase field model for modeling fracture – the isotropic second-order phase field fracture model – has been developed by Francfort and Marigo [11] by considering the regularized approximation given by Bourdin *et al.* [9]. Amor *et al.* extended it an anisotropic model, where the elastic energy was split into volumetric and deviatoric parts in order to prevent compressive loading cracks. A higher order phase field model proposed by Borden *et al.* [8], improved the computational cost of numerical methods. Karthik *et al.* [14] compared the phase field and gradient enhanced damage models. Kasirajan *et al.* [15] applied a C^1-continuous natural neighbor Galerkin method to the phase field models and showed its advantages over the standard finite element method. An extension to solve the dynamic loading in brittle materials using a hybrid phase field method [1] was studied by Raghu *et al.* [17]. We also note an exhaustive literature on finite element methods used in material phase field models given in [18].

Although the phase field method has been used by several authors for predicting crack propagation, few of the standard functions are improvised to match the actual behaviour. The degradation function for the strain energy $g(\phi)$ was taken as a linear $(1-\phi)$ or quadratic function $(1-\phi)^2$. But, in order to correct the energy degradation in actual computations, we have considered a higher order polynomial — viz. $(1-10\phi^3+15\phi^4-6\phi^5)$. The use of this polynomial can be better explained by Fig. 1. It is known that for lower damage values located in the interval $[0, 0.3]$, the degradation functions used in literature show up to 50% in the energy reduction. It is not possible in actual scenario since only 10% of energy can be reduced up to this point. However, such dynamics is correctly represented by the degradation function chosen in this paper. A hybrid formulation is employed where the computational time is reduced by considering the history parameter H in the phase field evolution equation and the linear balance of momentum equation is retained for solution of displacements. This also leads to the effective use of staggered algorithm for solving the phase field method.