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Dispersion-Managed Lump Waves in a Spatial Symmetric KP Model

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Abstract. This paper aims to explore dispersion-managed lump waves in a spatial symmetric KP model. Negative second-order linear dispersive terms play an important role in creating lump waves with the nonlinearity in the model. The starting point is a Hirota bilinear form with an ansatz on quadratic function solutions to the corresponding Hirota bilinear equation. Symbolic computation with `Maple` is conducted to determine lump waves, and characteristic behaviors are analyzed for the resulting lump wave solutions.

AMS subject classifications: 35Q51, 35Q53, 37K40

Key words: Soliton, lump wave, Hirota bilinear derivative, symbolic computation, dispersion.

1. Introduction

Generally speaking, the amplitudes and widths of waves change during propagation in nonlinear media. Under certain circumstances, however, the effects of nonlinearity and dispersion can cancel each other to create permanent and localized waves called solitons. Such a kind of phenomenon was first observed in water waves [50, 51] and then in optical fibers [47].

In mathematical physics, there are a few methods to determine solitons in nonlinear dispersive models, two of which are the inverse scattering transform [2] and the Hirota bilinear method [8]. The inverse scattering transform can be used to solve Cauchy problems

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of integrable models [63] and obtain long-time asymptotics of solitonless waves [1]. We will apply the Hirota bilinear method to our analysis on lump waves in (2+1)-dimensions below.

Suppose that P is a polynomial in two space variables x, y and time t. A Hirota bilinear differential equation in (2+1)-dimensions is defined as follows:

$$P(D_x, D_y, D_t)f \cdot f = 0,$$

where D_x, D_y and D_t are Hirota's bilinear derivatives [8]

$$\begin{aligned} &D_x^p D_y^q D_t^r f \cdot f \\ &= \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^p \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^q \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^r f(x,y,t)f(x',y',t')\Big|_{x'=x,y'=y,t'=t} \end{aligned}$$

for nonnegative integers p, q, r. An associated partial differential equation (PDE) with a dependent variable u is often determined by some logarithmic derivative transformation of

$$u = 2(\ln f)_x, \quad u = 2(\ln f)_{xx}, \quad u = 2(\ln f)_{xy}.$$

Within the Hirota bilinear theory, an N-soliton solution (please refer to, e.g., [7,25,26,39]) is presented through

$$f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij}\right),$$

where $\sum_{\mu=0,1}$ stands for the sum over all possibilities for $\mu_1, \mu_2, \cdots, \mu_N$ taking either 0 or 1, and the wave variables and the phase shifts are defined by

$$\begin{aligned} &\xi_i = k_i x + l_i y - \omega_i t + \xi_{i,0}, && 1 \le i \le N, \\ &\mathrm{e}^{a_{ij}} = -\frac{P(k_i - k_j, l_i - l_j, \omega_j - \omega_i)}{P(k_i + k_j, l_i + l_j, \omega_j + \omega_i)}, && 1 \le i < j \le N. \end{aligned}$$

In the above N-soliton solution, the wave numbers k_i, l_i and the frequencies ω_i, $1 \le i \le N$, are required to satisfy the associated dispersion relations

$$P(k_i, l_i, -\omega_i) = 0, \quad 1 \le i \le N,$$

but the phase shifts $\xi_{i,0}$, $1 \le i \le N$, are arbitrary constants. There are abundant applications of the Hirota bilinear method to nonlinear dispersive wave equations [10,34].

Lump waves (or rogue waves) in integrable models are remarkably varied, and they can describe diverse nonlinear phenomena [43]. Such waves are determined by means of rational functions, and localized in all directions in space [43, 44, 54]. Computing long wave limits of solitons can also lead to lump wave solutions [52]. It is known that the KPI equation has diverse lump wave solutions [19], and its special lump waves can be generated from its solitons, indeed [45]. Other integrable models which possess lump waves include the three-wave resonant interaction [12], the Davey-Stewartson II equation [52],