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An Adaptive Finite Element DtN Method for Maxwell's Equations

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Dedicated to Professor Tao Tang on the occasion of his 60th birthday.

Abstract. We consider a numerical solution to the electromagnetic obstacle scattering problem in three dimensions. Based on the Dirichlet-to-Neumann (DtN) operator, the exterior problem is reduced into a boundary value problem in a bounded domain. An a posteriori error estimate is deduced to include both the finite element approximation error and the DtN operator truncation error, where the latter decays exponentially with respect to the number of truncation terms. The discrete problem is solved by the adaptive finite element method with the transparent boundary condition. The effectiveness of the method is illustrated by numerical experiments.

AMS subject classifications: 78A45, 78M10, 65N30, 65N12, 65N50

Key words: Maxwell's equations, electromagnetic scattering problem, DtN operator, transparent boundary condition, adaptive finite element method, a posteriori error estimate.

1. Introduction

Scattering problems are concerned with the interaction between an inhomogeneous medium and an incident field. They have significant applications in many scientific areas including geophysical exploration, non-destructive testing, and medical imaging [13]. Motivated by significant applications, scattering problems have received great attention in both of the engineering and mathematical communities. A considerable amount of mathematical and numerical results are available for the scattering problems of acoustic, elastic,

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and electromagnetic waves. We refer to the monographs [21, 28, 30] on comprehensive accounts of the electromagnetic scattering theory for Maxwell's equations.

In this paper, we consider a numerical solution to the electromagnetic obstacle scattering problem in three dimensions. In addition to the large scale computation of the three-dimensional problem, there are two other main challenges: the scattering problem is imposed in an unbounded domain and the solution may have local singularity due to the nonsmooth surface of the obstacle. The first issue is concerned with the domain truncation where a transparent boundary condition is preferred to avoid artificial wave reflection; the second difficulty can be resolved by using the adaptive finite element method to balance the accuracy and computational cost.

One of the most popular methods for domain truncation is the perfectly matched layer (PML) technique, which was proposed by Bérenger to simulate the electromagnetic wave propagation in unbounded domains [6]. The idea of PML is to put a layer of artificially absorbing media around the computational domain so that outgoing waves can be attenuated. Mathematically, it was proved in [12] that when the thickness of the layer is infinity, the PML solution in the domain of interest is the same as the solution of the original scattering problem. However, in practice, the layer needs to be truncated to finite thickness which inevitably introduces the truncation error. The overall error contains three parts when applying the finite element method to the PML problem: the truncation error of the PML layer, the discretization error in the PML layer, and the discretization error in the domain of interest. It was shown in [3] that the PML truncation error decays exponentially with respect to the thickness of the layer and the PML parameters. As is known, the artificial PML layer is constructed through the complex coordinate stretching [11], which makes the PML layer to be an inhomogeneous medium. It is difficult to balance the efficiency and accuracy if a uniform mesh refinement is used. If a thin PML layer is used to reduce the computational cost, then the discretization error is large since the medium is inhomogeneous in the layer; on the contrary, if the discretization error is controlled to be small, then a thick PML layer is preferred, which increases the cost. To handle this issue, the adaptive finite element method is effective, especially when combined with a posteriori error estimates. Based on numerical solutions, the a posteriori error estimates can be used for mesh modification such as refinement or coarsening [32]. The method can control the error and asymptotically optimize the approximation. Moreover, it can effectively deal with the issue that the solution has local singularities in the domain of interest. It is worth mentioning that even though the solution is smooth, the adaptive finite element method is still desirable due to the inhomogeneous medium in the PML layer. We refer to [8–10, 16, 18] for the discussion of adaptive finite element PML methods for scattering problems in different structures.

Another effective approach is to impose transparent boundary conditions to solve the scattering problems formulated in open domains. A key step of the method is to construct the Dirichlet-to-Neumann operator, which can be done via different manners such as the boundary integral equation [14], the Fourier transform or Fourier series expansions [17, 22]. In this paper, observing that the solution is analytical when it is away from the obstacle, we consider the Fourier series expansion of the solution on any sphere en-