East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.020310.120410a

Vol. **1**, No. 1, pp. 20-34
February 2011

Memory-Reduction Method for Pricing American-Style Options under Exponential Lévy Processes

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Received 2 March 2010; Accepted (in revised version) 12 April 2010

Available online 26 October 2010

Abstract. This paper concerns the Monte Carlo method in pricing American-style options under the general class of exponential Lévy models. Traditionally, one must store all the intermediate asset prices so that they can be used for the backward pricing in the least squares algorithm. Therefore the storage requirement grows like $\mathcal{O}(mn)$, where m is the number of time steps and n is the number of simulated paths. In this paper, we propose a simulation method where the storage requirement is only $\mathcal{O}(m+n)$. The total computational cost is less than twice that of the traditional method. For machines with limited memory, one can now enlarge m and n to improve the accuracy in pricing the options. In numerical experiments, we illustrate the efficiency and accuracy of our method by pricing American options where the log-prices of the underlying assets follow typical Lévy processes such as Brownian motion, lognormal jump-diffusion process, and variance gamma process.

Key words: American options, Monte Carlo simulation, memory reduction, exponential Lévy processes.

1. Introduction

During the past decade, the exponential Lévy models have been popularized in financial modeling among researchers as well as practitioners, see e.g. [11]. The classical Black-Scholes model [3] presumes that the price of the underlying asset follows a geometric Brownian motion with constant volatility. However, the empirical observation in real financial trading reveals that the implied volatility surface often displays a so-callel volatility smile [18]. Moreover, the distribution of the asset return, assumed to be Gaussian in the Black-Scholes model, exhibits a heavy tail [10], i.e. large moves of the market

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have decent probabilities to occur. As remedies for Black-Scholes, the exponential Lévy models contain Lévy jumps in addition to the classical diffusion, so that the phenomena of the volatility smiles and the heavy tails can be generically accounted for [11]. We remark that the exponential Lévy model is a very general class of models. It includes well-known examples such as the Black-Scholes model [3], lognormal jump-diffusion model [19], double-exponential jump-diffusion model [15], variance gamma model [17], normal inverse Gaussian model [2], CGMY model [5], etc. We refer to the classical reference [11] for further background in financial modeling by exponential Lévy processes. The present paper concerns the use of Monte Carlo simulation in pricing American-style options under the general framework of exponential Lévy models.

It is well known, see e.g. [13], that with the no-arbitrage principle the option price is given by the discounted expected payoff under certain risk-neutral measure. This leads to option pricing by the Monte Carlo method, for which the first application was made by Boyle [4] in 1977. Since then, Monte Carlo method has been a popular tool in pricing financial derivatives [13]. Yet, Monte Carlo method is known to have difficulties in handling American-style options with early exercise feature. In 2001 Longstaff and Schwartz [16] proposed a practical algorithm, named least squares method (LSM), to price American options. Their method is based on a backward-in-time induction, where at each time step the continuation value of the option is estimated by a least square approximation.

However, one drawback of LSM is that, in order to compute the intermediate exercise prices at all time steps, it requires the storage of all asset prices at all time steps for all simulated paths. Thus the total storage requirement grows like $\mathcal{O}(mn)$ where m is the number of time steps and n is the number of simulated paths. The plain Monte Carlo method, referred as the *full-storage method* in this paper, is therefore computationally inefficient since the accuracy of the simulation is severely limited by the storage requirement.

This storage problem can be alleviated by "bridge methods" such as the Brownian bridge [9], the inverse Gaussian bridge [22], and the gamma bridge [23] — where the memory requirement can be reduced to $\mathcal{O}(n \log m)$. Nevertheless, one drawback is that a specific bridge method can only work on the corresponding model that the price of the underlying asset follows. Thus the Brownian bridge is suitable for Brownian motion, the gamma bridge for the variance gamma process, and so on. That is to say, all bridge methods are model-dependent, which limits their use in applications.

In this paper, we develop a memory-reduction method, which does not require storing of all intermediate asset prices. The storage is significantly reduced to $\mathcal{O}(m+n)$. Coupled with the least squares method proposed in [16], our memory-reduction method is applicable to the general class of exponential Lévy processes. The main idea of our method is to first generate the price process forward until the expiration time, and to store only the *seeds* of the random number sequences at each time step. When computing the option prices backwardly, we recompute the just-in-time asset prices using the corresponding seeds. Since the prices are recomputed exactly, the memory-reduction method gives the same result as the full-memory method. The additional computational cost is the cost of regenerating the random numbers corresponding to the asset prices. The total computational cost is therefore always less than twice that of the full-storage method.