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Mathematical Representation of Social Capital and Human Bonds shown for the Great East Japan Earthquake

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Abstract. Directly after the Great East Japan disaster, various acts for mutual aid among victims and also between victims and non-victims spread out in different places throughout Japan. This paper presents a mathematical formulation of social capital regarded as existing as the background to these actions, with application to refuge phenomena in the aftermath of the accident at the Fukushima Daiichi Nuclear Plant.

AMS subject classifications: 91C99

Key words: Social capital, disaster, mathematical model.

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1. Introduction

On March 11, 2011 at 1:30 pm, the first author[†] and his wife finished golf at their home course. They went down, as usual, to a restaurant near a fishing port. After lunch they left for home along a coastal road, and at 2:46 pm an earthquake struck as they were driving. They stopped the car and waited for it to end. During that time, houses at the road-side fell and telephone poles shook. The spectacle appeared to be one taken right out of a movie.

Around 3:00 pm, they set off for home again. The car radio warned of an impending tsunami, and they had to choose either to leave for home by the seaside or to go up into the hills. In an instant, they chose the second alternative, which separated life from death for them. If they had chosen to return to their house, they might have been engulfed by the tsunami. Around 3:15 pm, the tsunami struck the shore, creating a dreadful panorama. When the tsunami had ebbed away, cars, huts and other debris were drifting and being

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carried away offshore. Fortunately, their house remained largely intact, but the interior of the first floor was completely destroyed by the waves.

For a few days thereafter, they stayed at the gymnasium of a high school as a place of refuge. A warm-hearted exchange of human feelings, which reminded them of the Japanese old and good life, arose among the people gathered there. It was a valuable experience for them.

As described in this paper, we explore the question of "How is life space filled with the exchange of human feelings and the rule of reciprocity created in the extreme situation described above?" We attempt to build a mathematical model describing it. The fundamental idea to address the question is to use the concept of social capital, together with the study of steps to address regional regeneration from economic depression. The concept of social capital is presented in the next section.

2. Social Capital

2.1. History of social capital

The conceptual origin of social capital appeared in the 1910s, during the World War I era. When the traditional sense of cooperation and public goodness (welfare, common weal) prevailing in village communities was collapsing with industrialisation and urbanisation rising in America in that era, the education specialist Hanifan discussed the situation from the perspective of social capital [1]. Putnam made Hanifan's approach famous by developing his results [2,3]. Putnam inquired, "Why do democratic governments succeed or fail in carrying out their policies?" In particular, he explained the difference of performance in administration of local governments in Italy, using the concept of social capital based on the accumulation of case studies on that question over a period of 20 years. Theories and applications of the social capital concept have since progressed in fields such as politics, economy, social welfare, environmental sanitation, education, and criminology [4,5].

2.2. Definition of social capital

Regional societies have confronted various difficulties including aging, a declining birthrate, concurrent worldwide recession, and extension of production abroad. Some have succeeded in planning strategies to invest in human resources, the environment, and relationships among inhabitants in places of public enterprise. The common phenomenon is to use the power of community effectively. The fundamental source of power to act is social capital — a community rich in social capital has sufficient possibilities to solve difficult problems. Social capital is defined by an accumulation of networks, built up and based upon reliability and reciprocity among community members. "In the community linked together with reliability, people take action with others, expecting to receive a return in future. By repeating these exchanges, reciprocity is developed. And then the reciprocity is raised to a kind of social precept with time. That becomes a model of good behaviour."

2.3. Loss of social capital from the Fukushima Daiichi Nuclear Power Plant accident

Radioactive materials in various places at Fukushima so far have not injured humans, but illusions about radioactivity are eroding human confidence and consciousness because the risks posed by emissions are not clearly communicated to people. Significant questions are:

- What threat does low-level radioactivity remaining in living environments pose for the health of the inhabitants over several generations?
- What is the threshold of radiation exposure below which they are safe?

Nobody has effectively communicated credible answers to such matters, and there are even two quite opposite positions — one is that we are safe because there is no proof of danger, and the other is that there is danger because there is no proof of safety. People are separated into factions by such differences, which clearly engender a loss of social capital in the community.

3. Mathematical Model to Describe Social Capital

3.1. Role of order parameter

Let us now presume that everything that brings people together (family, community, nation etc.) would be lost. Taylor proposed that a new common language such as one he expressed as "Personal Resonance" (PR) is necessary [6]. To realise his PR, an order parameter $k = k(x,t)$ is introduced to show a frequency in people composing the community, where $\{x \in R;\ 0 < x < N\}$ denotes the people in the community and t $(t > 0)$ the time, and N is the total population of the community. The interval for the parameter k is $0 < k < 1$, and $k = 1/2$ corresponds to the best PR frequency.

Hypothesis 1. A double well potential function $G(k)$ is set up as a mathematical formulation giving rise to barriers among members of community.

The wall that lies in the middle between two wells in the $G(k)$ has the function of separating two members, and the degree of harmony between them depends upon the wall height. The wall height is defined by a monotonic decreasing function, with a variable that is the sum of reciprocity and voluntary action as defined later. In addition, the height of the wall may take a negative value that affects assimilation of the two members into the state $k = 1/2$.

3.2. Equation describing the bonds in the community

Based on the notion of of social capital, we introduce the following evolution model involving the derivative $dG(k)/dk$ of the double well potential as a source term:

$$\begin{cases} \dfrac{\partial k}{\partial t} = -U(x)\dfrac{\partial k}{\partial x} + \nu \dfrac{\partial^2 k}{\partial x^2} - \dfrac{dG(k)}{dk}\,, & (t > 0\,,\ 0 < x < 1) \\ k(0,x) = \chi(x)\,, & (0 < x < 1) \\ \dfrac{\partial k}{\partial x}(t,0) = \dfrac{\partial k}{\partial x}(t,1) = 0\,, & (t > 0), \end{cases} \tag{3.1}$$

where we adopt $G(k) = 16h(s)k^2(k-1)^2$ $(0 < k < 1)$ with $h(s) = H - s$ $(0 < s < 2H)$. There a similar model description in the authors' book [7], where the time evolution of coastal ecosystems is based on a phase field model and the optimization method of genetic algorithms.

The initial condition

$$k(x) = \begin{cases} 1\,, & x > 0 \\ 0\,, & x \le 0 \end{cases}$$

means that the frequencies in the community are divided into two different states — i.e. the degree of human bonding is weak. The partial differential equation equation in the model (3.1) describes the time evolution of the order parameter — i.e. personal resonance or the human bond. Here $U(x)$ represents an advection effect that enhances the inner circulation of the community, and H denotes the reciprocity height. Further, we assume that $h = h(R(t) + A(t))$, and in subsection 3.4 specify H, $R(t)$ and $A(t)$.

Fig. 1 illustrates some typical behaviour of the personal resonance parameter k for different sets of h and $U(x)$, where it is notable that a lower reciprocity height H brings about an earlier maximum personal resonance. We note that the advection effect for the type of nonzero $U(x)$ assumed does seem to produce locally stable states.

3.3. Mathematical model describing the rule of reciprocity

We adopt

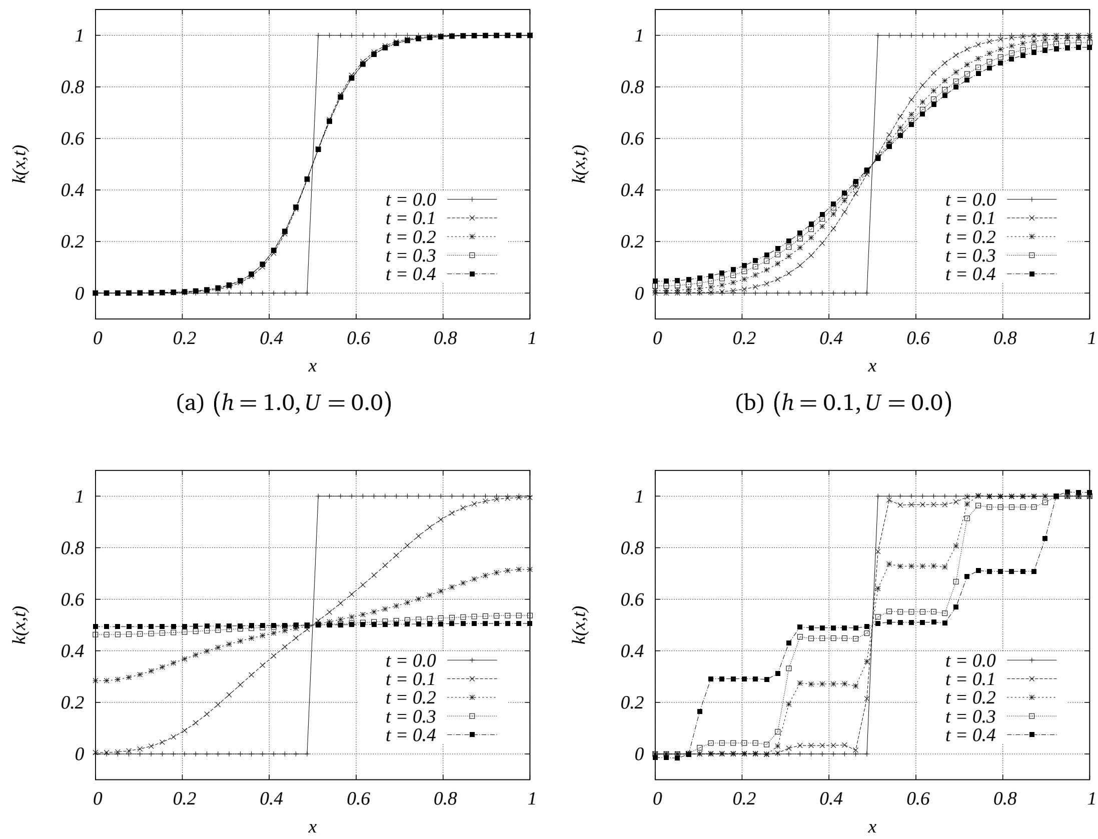
$$R = R(t)\,, \qquad t > 0$$

as the level of present return, and

$$A = A(t)\,, \qquad t > 0$$

as the level of voluntary action. As time passes, the level of reciprocity increases in proportion to the level of voluntary action. Similarly, the level of voluntary action increases in proportion with the level of reciprocity. Furthermore, constant terms to represent latent factors they each involve are incorporated. Thus we choose to adopt the following system of simultaneous differential equations for $t > 0$:

$$\frac{dR}{dt} = p(A - aR) + E\,, \qquad \frac{dA}{dt} = q(R - bA) + F\,, \tag{3.2}$$



(a) $(h = 1.0, U = 0.0)$

(b) $(h = 0.1, U = 0.0)$

(c) $(h = -1.0, U = 0.0)$

(d) $(h = -1.0, U(x) = 10 \sin 10\pi x)$

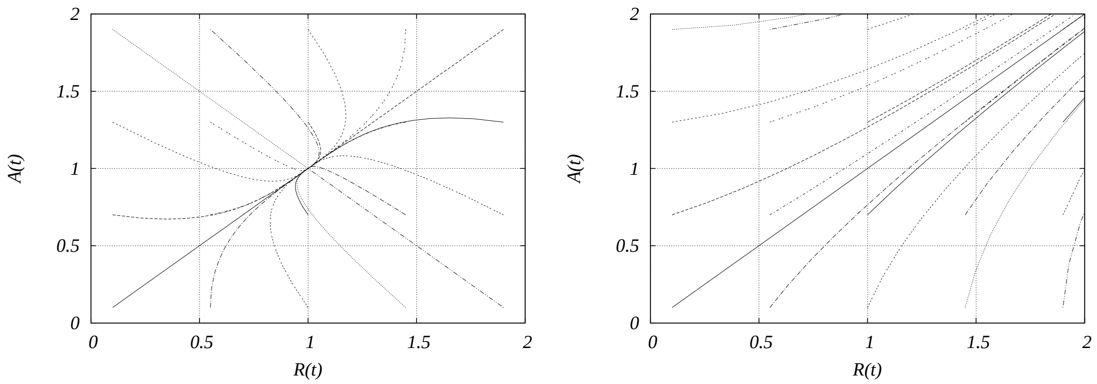
Figure 1: Behaviour of personal resonance for different parameters.

where a, b, p, and q are all positive constants and a sufficient condition for the existence of an equilibrium point is $ab > 1$. The latent factors E and F have a definite meaning. In Ref [8], there is the statement "One does not help one another from friendship and confidence, but one helps one another because he believes that it does good for him in future to continue the relation". This has been referred to as the "Shadow of the Future" [9].

If we solve the system (3.2) under suitable initial conditions such as $A(0) = 0$ and $R(0) = 0$, for $ab > 1$ the solution converges to the equilibrium point

$$R^{\infty} = \left(\frac{F}{q} + \frac{bE}{p}\right) / (ab - 1) , \qquad A^{\infty} = \left(\frac{E}{p} + \frac{aF}{q}\right) / (ab - 1) , \tag{3.3}$$

where the coefficients p and q define the speed to form the rule of reciprocity in the community, and $H = A^{\infty} + R^{\infty}$ is the reciprocity height mentioned earlier. Fig. 2 shows reciprocity behaviour for the cases $ab > 1$ and $ab < 1$, where $p = q = E = F = 1.0$.



(a) $(a = b = 2.0)$ (b) $(a = b = 0.5)$

Figure 2: Reciprocity behaviour for two cases where $ab > 1$ and $ab < 1$, respectively.

3.4. Index of social capital

For a solution $k(x,t)$ $(0 < x < 1, t > 0)$, we consider the index of social capital $(t > 0)$

$$K(e,t) = M\left\{x \,\middle|\, \frac{1}{2} - e < k(x,t) < \frac{1}{2} + e\right\} \tag{3.4}$$

and also write

$$L(e,t) = 1 - K(e,t)\,, \tag{3.5}$$

where M means the measure in $(0,1)$ and the parameter e $(0 < e < 1/2)$ signifies a control to show whether or not social capital is created easily in the community. If the value of e is small, then social capital is difficult to create. If the index $K(e,t)$ is large, then the community is regarded as rich in social capital.

4. Behaviour of the People of Iwaki during the Accident at the Fukushima Daiichi Nuclear Power Plant

Around 3 pm on March 13, 2011: The first building of Fukushima Daiichi Nuclear Power Plant blew up with a terrible explosion.

Around 11 am on March 14, 2011: The building of the third plant housing a reactor blew up. Nuclear fuel rods were exposed to air and the cooling system stopped.

March 15, 2011: At Fukushima Daiichi Power Plant, the situation had become difficult to control, but it worsened. In fact, the fourth plant building, which had been stable, also had an explosion. It was impossible to do repair work because of the great amount of radiation emissions in the surrounding yard. The second plant building, in which the nuclear fuel rods were stored, exploded. Pressure control systems were damaged.

As things stood, groundless rumors were spreading in the Iwaki district. At the time people were already talking about the meltdown of nuclear fuel rods in the reactor, which was confirmed by inspection later. From around March 15, people started to leave Iwaki and Fukushima. It would seem that many people ran away. However, most people who took refuge returned home by mid-April, and those who remained in the refuge areas were very few. To illustrate the circumstances from the perspective of social capital, we classify the people of the district into three groups ($t > 0$):

1. People who remain in their homes, numbered as $S = S(t)$;
2. People who stay in the refuge areas. numbered is $I = I(t)$;
3. People who return home from the refuge areas, numbered as $R = R(t)$.

The behaviour is then described by the following model ($t > 0$):

$$\frac{dS}{dt} = -mSI \,,$$
$$\frac{dI}{dt} = mSI - nI \,,$$
$$\frac{dR}{dt} = nI \,, \tag{4.1}$$

where the well-known SIR model [10] is adopted.

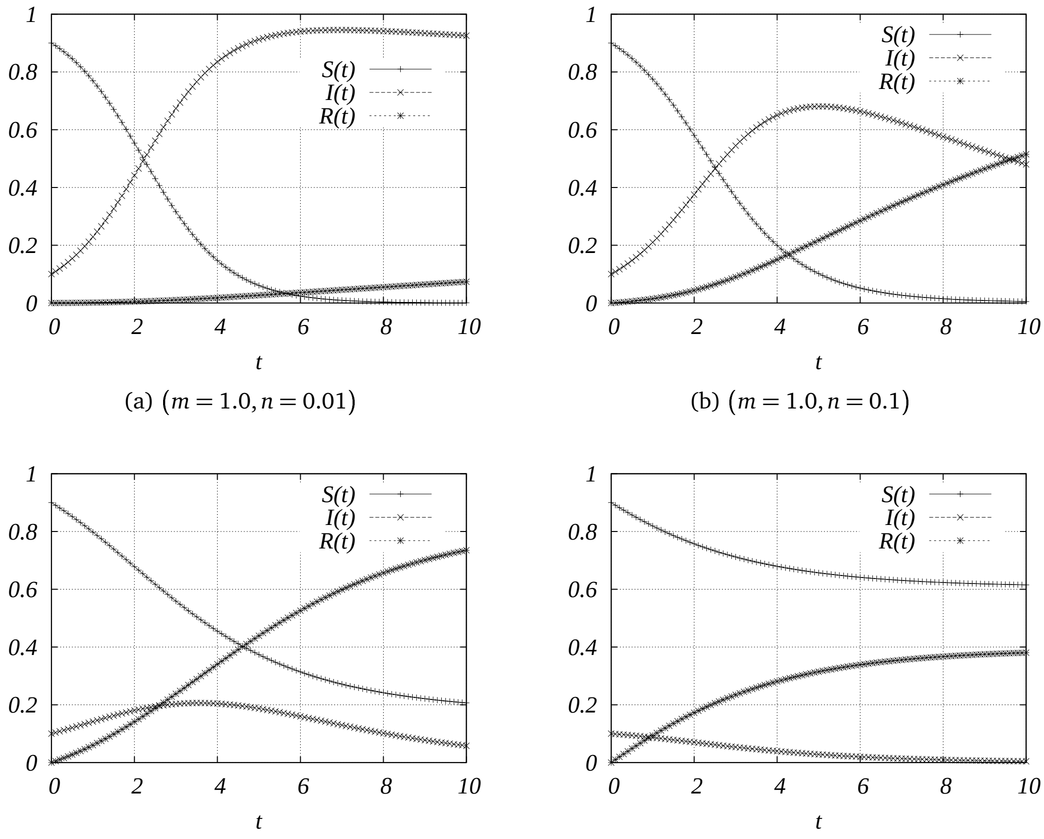
The parameters m and n depend upon social the capital cultivated in the district and denote the coefficients of refuge and return, respectively. If social capital is rich, then the coefficient m might be small — in fact, let $m = L(e)$. Similarly, we let $n = K(e)$ as people who live in a district with plenty of social capital tend to come home. Some properties of the solution are:

1. $I + S + R = S(0) + I(0) = N$; and
2. $I = N - S + (m/n)\log(S/S(0))$.

Moreover, S is monotonously decreasing, so I takes the value 0 at a certain $t = t'$. We write $S(t') = S'$, and some results from a numerical solution for $\{I, R, S\}$ are illustrated in Fig. 2.

5. Mathematical Interpretation for Loss of Social Capital

Let us attempt to explain a process of dividing the community into fractions based on radioactivity. We might assume that the coefficients a and b of the control term in the system (3.2) are sufficiently large, and also the terms E and F showing a shadow of the



(a) $(m = 1.0, n = 0.01)$ (b) $(m = 1.0, n = 0.1)$

(c) $(m = 1.0, n = 0.5)$ (d) $(m = 1.0, n = 1.0)$

Figure 3: S, I, and R for different parameters.

future are sufficiently small. The height of reciprocity is then fairly low, compared with the height of reciprocity in a standard community. If the values above reflect the framework for a standard community, the height of the double well potential is still high. The value of $K(e)$ is rather small, which implies a loss of social capital.

6. Conclusion

The importance of social capital in the community was discussed by building mathematical models. The loss of social capital brought about by cutting people off in the community should be avoided to the greatest degree possible. In the Fukushima area struck by the tsunami, it is planned to construct a city of the future on higher ground. The city should be designed to be both efficient and rich in social capital. The mode of design should provide for industrial accumulation, in which the linkages between industries work smoothly and an un-traded interdependency exists. To facilitate the accomplishment of such a design, we intend to produce a mathematical model that describes that plural bonds at work in the community.

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