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Sinc Nyström Method for Singularly Perturbed Love's Integral Equation

Fu-Rong Lin[1,*], Xin Lu[1] and Xiao-Qing Jin[2]

[1] *Department of Mathematics, Shantou University, Shantou, Guangdong, 515063, China.*
[2] *Department of Mathematics, University of Macau, Macao, China.*

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Abstract. An efficient numerical method is proposed for the solution of Love's integral equation

$$f(x)+\frac{1}{\pi}\int_{-1}^{1}\frac{c}{(x-y)^2+c^2}f(y)dy=1, \quad x\in[-1,1]$$

where $c>0$ is a small parameter, by using a sinc Nyström method based on a double exponential transformation. The method is derived using the property that the solution $f(x)$ of Love's integral equation satisfies $f(x)\to 0.5$ for $x\in(-1,1)$ when the parameter $c\to 0$. Numerical results show that the proposed method is very efficient.

AMS subject classifications: 45L10, 65R20

Key words: Love's integral equation, sinc function, Nyström method, DE-sinc quadrature.

1. Introduction

We consider numerical methods for the solution of Love's integral equation

$$f(x)+\frac{1}{\pi}\int_{-1}^{1}\frac{c}{(x-y)^2+c^2}f(y)dy=1, \quad x\in[-1,1], \tag{1.1}$$

where $c>0$ is a small parameter. This integral equation arises in determining the capacity of a circular plate condenser, and it has been shown to possess a unique, continuous, real and even solution [5].

Different numerical methods for the solution of (1.1) have been proposed by several authors. The equation with $c=1$ was considered in Refs. [2–4, 17, 18]. Agida & Kumar proposed a solution scheme for $c\geq 1$, based on Boubaker polynomials [1]. For $c<1$,

*Corresponding author. *Email addresses:* frlin@stu.edu.cn (F.-R. Lin), xqjin@umac.mo (X.-Q. Jin)

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there are numerical difficulties [6, 10–12]. Numerical results have been presented by Pastore [10] for small $c > 0$ — viz. $c \in [10^{-4}, 10^{-2}]$. In this article, we consider even smaller values — i.e. $c \leq 10^{-7}$.

We derive our method by exploiting the property that for $x \in (-1, 1)$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{c}{(x-y)^2+c^2}f(y)dy \to f(x)$$

when $c \to 0$ — i.e. the solution of (1.1) is nearly equal to $1/2$ for $x \in (-1, 1)$ [6]. We discretise the integral equation by using a DE-sinc quadrature, which is a sinc quadrature based on a double exponential (DE) transformation. The DE transformation was first proposed by Takahasi and Mori [16] for an efficient evaluation of integrals of analytic functions with singularities at end-points, and it is useful not only for numerical integrations but also for various kinds of sinc numerical methods [13, 15]. Ref. [8] provides a review.

The outline of the remainder of this article is as follows. In Section 2, we summarise some basic results for sinc approximations and DE transformations, and a DE-sinc quadrature that is then applied to Love's equation (1.1) in Section 3. Numerical results in Section 4 illustrate the efficiency and accuracy of the proposed numerical scheme.

2. A DE-sinc Quadrature

There are some basic results for sinc numerical methods based on double exponential transformations, or socalled DE-sinc numerical methods. In particular, we introduce a DE-sinc quadrature. Let us first mention some familiar related notation and concepts:

- The set of all integers, the set of all real numbers, and the set of all complex numbers are denoted by $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$, respectively;
- x and z denote the real and complex variables, respectively; and
- D_d is the strip region of width $2d$ ($d > 0$) defined by

$$D_d = \{\zeta \in \mathbb{C} : |\mathrm{Im}\,\zeta| < d\}\,.$$

The sinc function is defined by

$$\mathrm{sinc}(x) = \begin{cases} \dfrac{\sin(\pi x)}{\pi x}, & x \neq 0\,, \\ 1, & x = 0\,. \end{cases}$$

Let $h > 0$ denote the mesh size in the sinc approximation, and let

$$S_{k,h}(x) \equiv \mathrm{sinc}(x/h - k)\,, \quad k \in \mathbb{Z}$$

denote the sinc bases corresponding to h.