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Fast Exponential Time Integration for Pricing Options in Stochastic Volatility Jump Diffusion Models

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Abstract. The stochastic volatility jump diffusion model with jumps in both return and volatility leads to a two-dimensional partial integro-differential equation (PIDE). We exploit a fast exponential time integration scheme to solve this PIDE. After spatial discretization and temporal integration, the solution of the PIDE can be formulated as the action of an exponential of a block Toeplitz matrix on a vector. The shift-invert Arnoldi method is employed to approximate this product. To reduce the computational cost, matrix splitting is combined with the multigrid method to deal with the shift-invert matrix-vector product in each inner iteration. Numerical results show that our proposed scheme is more robust and efficient than the existing high accurate implicit-explicit Euler-based extrapolation scheme.

AMS subject classifications: 91B28, 62P05, 35K15, 65F10, 65M06, 91B70, 47B35

Key words: Stochastic volatility jump diffusion, European option, barrier option, partial integro-differential equation, matrix exponential, shift-invert Arnoldi, matrix splitting, multigrid method.

1. Introduction

In 1973, Black and Scholes [4] inytoduced a model to compute the price of a European option in the finance industry. Later empirical evidence indicated that the model assumptions on log-normality of the return of the underlying asset and constant volatility are usually inconsistent with market prices [24, 29]. Several extensions of the Black-Scholes model have been proposed. Examples include the jump diffusion [19, 24] or pure jump Lévy models [2, 6, 7, 11, 23], the stochastic volatility (SV) model [15, 18], the stochastic volatility with jumps in return (SVJ) model [3], and the stochastic volatility with correlated and contemporaneous jumps in return and variance (SVCJ) model [10, 13]. Of these, the SVCJ model usually offers a much better match with market prices than the others, since

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it allows the volatility to be stochastic and can capture the jumps in both the return and variance very well. In this article, we consider option valuation in the SVCJ model.

One way to price options involves solving a partial integro-differential equation (PIDE). For the SVCJ model, the corresponding PIDE is a two-dimensional equation involving a convolution integral defined over an infinite domain. D'Halluin *et al.* [9] proposed a second-order Crank-Nicolson scheme and Rannacher time-stepping to approximate PIDE, but direct application of the Crank-Nicolson scheme to the PIDE arising in the SVCJ model would suffer from the inversion of a block dense matrix at each time step. Andersen & Andreasen [1] used an operator-splitting approach combined with the fast Fourier transform (FFT) evaluation of a convolution integral to price European options with jump diffusion, but their method cannot easily handle the multi-dimensional jump diffusion processes and processes with state-dependent jump magnitude distributions — cf. [14]. Recently, an extrapolation scheme based on the implicit-explicit (IMEX) Euler method was proposed by Feng & Linetsky [14], to deal with the jump-diffusion PIDE, where the differential term is treated implicitly for stability and the integral term explicitly for numerical efficiency. Combined with the extrapolation approach, this approach is remarkably fast and can achieve polynomial accuracy in the time direction. Zhang *et al.* [34] further improved the efficiency of the extrapolation scheme by coupling quadratic finite elements for spatial discretization with preconditioning techniques for the resulting systems. It is known that the IMEX Euler-based extrapolation scheme belongs to the class of time-stepping methods. When the time interval is large, many time steps may be required in order to achieve a given accuracy, which is very time consuming. Instead of using a time-stepping scheme, some other authors have proposed to exploit the exponential time integration (ETI) scheme to solve PIDEs arising from both the Black-Scholes and Merton models [28, 32]. The ETI scheme is a one-step method, and we need not consider stability nor temporal discretized accuracy.

This article discusses the application of the ETI scheme for pricing options in the SVCJ model, such that the price of an option involves the product of a matrix exponential and a vector. In Refs. [28,32], the corresponding matrix exponential was directly computed by a scaling and squaring algorithm with Padé's approximation [16], which has $O(n^3)$ complexity where n is the matrix size, whereas we need to find the product of a matrix exponential and a vector but not the exact matrix exponential where the recently developed Krylov subspace methods often work very well [12, 17, 22, 25, 33]. Quite recently, Lee, Liu & Sun [20] employed the shift-invert Arnoldi method proposed in Ref. [21], to implement the ETI scheme for option pricing in the Merton [24] and Kou jump-diffusio [19] models. By exploiting the Toeplitz structure of the resulting matrix and the Gohberg-Semencul formula (GSF) for inversion of the Toeplitz matrix, the computational cost of the ETI scheme was reduced dramatically to $O(n \log n)$ operations. It is notable that the PIDE arising in the SVCJ model has two space variables related to the asset price and the volatility, respectively. After the spatial discretization of the PIDE by central differences and the time integration of the semi-discretized ODE system by the ETI scheme, we obtain a solution vector as the product of the exponential of a block Toeplitz matrix and a vector. When the shift-invert Arnoldi method is applied to approximate this resulting solution vector, a