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Submatrix Constrained Inverse Eigenvalue Problem involving Generalised Centrohermitian Matrices in Vibrating Structural Model Correction

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Abstract. Generalised centrohermitian and skew-centrohermitian matrices arise in a variety of applications in different fields. Based on the vibrating structure equation $M\ddot{x}+(D+G)\dot{x}+Kx=f(t)$ where M,D,G,K are given matrices with appropriate sizes and x is a column vector, we design a new vibrating structure mode. This mode can be discretised as the left and right inverse eigenvalue problem of a certain structured matrix. When the structured matrix is generalised centrohermitian, we discuss its left and right inverse eigenvalue problem with a submatrix constraint, and then get necessary and sufficient conditions such that the problem is solvable. A general representation of the solutions is presented, and an analytical expression for the solution of the optimal approximation problem in the Frobenius norm is obtained. Finally, the corresponding algorithm to compute the unique optimal approximate solution is presented, and we provide an illustrative numerical example.

AMS subject classifications: 65F18, 15A51, 15A18, 15A12

Key words: Left and right inverse eigenvalue problem, optimal approximation problem, generalised centrohermitian matrix, submatrix constraint.

1. Introduction

Generalised centrohermitian and skew-centrohermitian matrices arise in a variety of applications in fields such as information theory, linear system or estimate theory, signal processing, the numerical solution of differential equations and Markov processes — e.g. see Refs. [5, 7, 9–12, 17, 18, 21]. Here we consider vibrating structures such as bridges, highways, buildings and vehicles that are generally characterised by a linear second-order differential system

$$M\ddot{x}+(D+G)\dot{x}+Kx=f(t),$$

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where x is a column vector and M, D, G and K are matrices of appropriate size representing the mass (usually a diagonal matrix), damping, gyroscopic and stiffness, respectively. The general solution to the corresponding homogeneous equation $M\ddot{x}+(D+G)\dot{x}+Kx=0$, on omitting the forcing function $f(t)$, plays an important role in the stability of the vibratory behaviour. In particular, we discuss the undamped non-gyroscopic model governed by

$$\begin{cases} M\ddot{x}+Kx=0\,, \\ \ddot{y}^{\mathrm{H}}M+y^{\mathrm{H}}K=0\,, \end{cases}$$

where y is a column vector of the same size as x and superscript H denotes the conjugate transpose (cf. below). The relevant solution form

$$\begin{cases} x(t)=\mathbf{u}\,e^{\lambda t}\,, \\ y(t)=\mathbf{v}\,e^{\mu t}\,, \end{cases}$$

for this linear system immediately leads to the two quadratic eigenvalue problems

$$\begin{cases} (\lambda^2 M+K)\,\mathbf{u}=0\,, \\ \mathbf{v}^{\mathrm{H}}(\mu^2 M+K)=0\,, \end{cases}$$

where $(\lambda,\mathbf{u})$ and $(\mu,\mathbf{v})$ are their eigenpair solutions, respectively. Purely imaginary eigenvalues ($\lambda=\mathrm{i}\lambda_1\,,\ \mu=\mathrm{i}\mu_1$) define the natural frequency (λ_1 or μ_1) of the system and the corresponding natural mode $\mathbf{u}\,(\mathbf{v})$. Letting $\widetilde{\lambda}=\lambda_1^2$, $\widetilde{\mu}=\mu_1^2$, $A=M^{-\frac{1}{2}}KM^{-\frac{1}{2}}$, $z_1=M^{\frac{1}{2}}\mathbf{u}$ and $z_2=M^{\frac{1}{2}}\mathbf{v}$, we have

$$Az_1=\widetilde{\lambda}\,z_1\,,\qquad z_2^{\mathrm{H}}A=\widetilde{\mu}\,z_2^{\mathrm{H}}\,. \tag{1.1}$$

The natural frequencies of the system and its associated natural modes are obviously determined by the stiffness matrix K or the mass matrix M. In practice, the stiffness matrix K is more complicated than the mass matrix M, and they are usually estimated by measurements or computed by some numerical methods (e.g. the finite element method). In engineering, some of the natural frequencies and natural modes can usually be identified in dynamic models, but there are often discrepancies between them and measured natural frequencies (natural modes). It is therefore often important to modify an approximate model such that the difference is minimised [13] — i.e. so the natural frequencies and natural modes in a corrected model are exactly the same as the identified natural frequencies and natural modes. In general, the stiffness or the mass matrix is corrected by vibration tests via nonlinear optimal optimisation techniques [3, 4], but the existence and the uniqueness of the solution and the solution is not always optimal. Here we present a method to correct such an approximation model based on the left and right inverse eigenvalue problem (with spectral and structural constraint), where we find a matrix A of order n containing the given part of left and right eigenvalues and corresponding left and right eigenvectors. Prototypes of this problem also arise in the perturbation analysis of matrix eigenvalues [19] and in recursive processes [8], and has practical application in scientific computation and other engineering fields.