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Traffic Lights or Roundabout? Analysis using the Modified Kinematic LWR Model

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Abstract. Traffic flow is treated as a continuum governed by the kinematic LWR model and the Greenshield flux function. The model is modified to describe traffic flow on a road with traffic lights or a roundabout. Parameters introduced determine the traffic flow behaviour and queue formation, and numerical simulations based on the Godunov method are carried out. The numerical procedure is shown to converge, and produces results consistent with previous analytic results.

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1. Introduction

Many roads in urban areas suffer from heavy traffic, and even on modern highways there can be "stop and go" traffic. In particular, traffic congestion often occurs in the approach to an intersection with traffic lights or a roundabout. Sometimes the congestion might be reduced by applying some new traffic regulation — e.g. by making a section of two-way road one-way, or by replacing traffic lights with a roundabout as discussed here. Simulations can help to assess whether the consequent modified traffic behaviour results in better traffic management. Traffic simulation packages have been developed since the 1960s [14], but research on traffic modelling and simulation is ongoing. Specific issues for traffic flow engineering include the study of highway junctions where there is merging [9, 10, 13], and the effects of entrances and exits [1]. The efficiency of a roundabout versus traffic lights at an intersection that we address is another.

Traffic may often be treated as a continuum in the macroscopic modelling of traffic flow. The kinematic Lighthill-Whitham-Richards (LWR) model [7,8] is often adopted, where the number of vehicles is assumed to be conserved and the flow is described in terms of traffic

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density and vehicular speed (flux). Driver behaviour is modelled by some specified prescribed flux to density relationship, and various such flux functions have been used [6].

The Godunov finite difference method [11, 12] with upwinding is an especially appropriate numerical approach when discontinuities are to be expected, and it is used here. Another procedure is the supply-demand method [3–5], which is also based on conservation of the number of vehicles. The analysis of traffic flow simulation using cellular automata for one-lane or multi-lane roundabouts has also been explored — e.g. see Refs. [2, 16, 17].

The rest of this article is organised as follows. In Section 2, the kinematic LWR model and its discrete formulation are discussed. In Section 3, we propose a mathematical model for traffic flow at an intersection with traffic lights or a roundabout, and numerical calculations are conducted to compare their relative performance. The length of the queue formed behind the intersection is discussed in Section 4, and our conclusions and discussion are in Section 5.

2. The Kinematic LWR model and its Discrete Formulation

We consider a road with heavy traffic moving in one direction. Let n vehicles per kilometre denote the traffic density (the number of vehicles in a unit length of road), and f vehicles per hour denote the flux (the number of vehicles passing in a unit length of time). The flux and density are related by

$$f = n \times v ,$$

where v is the mean velocity of the vehicles. In reality, the velocities depend on aspects such as individual driver characteristics, traffic density and road condition. However, here we adopt the commonly used Greenshield model, where the average velocity depends linearly on traffic density as follows:

$$v(n) = V_m\left(1 - \frac{n}{N_m}\right) , \tag{2.1}$$

where N_m (vehicles/km) and V_m (km/hour) are the maximum traffic density and velocity, respectively. This Greenshield velocity and flux are depicted in Fig. 1. In applications, the parameter V_m depends on road conditions — e.g. V_m is much larger on highways than along city or suburban roads. Although Eq. (2.1) is linear, it captures two important aspects of traffic flow — viz. when the road is nearly empty $n \to 0$ so the mean velocity tends to its maximum ($v(n) \to V_m$), whereas if the road is nearly full $n \to N_m$ so the mean velocity tends to zero ($v(n) \to 0$) when vehicles can hardly move. The underlying conservation principle that governs traffic dynamics is

$$n_t + f_x = 0 , \quad \text{with} \quad f(n) = nV_m\left(1 - \frac{n}{N_m}\right). \tag{2.2}$$

In this model, the maximum flux is given by the ordinate of the vertex parabola $f(n)$, which is $f_m = N_m V_m/4$. Eq. (2.2) can be rewritten $n_t + f'(n)n_x = 0$, where $f'(n)$ is the signal speed. As indicated in Fig. 1 (right), the signal speed is positive when $n(x,t) < N_m/2$,