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An Efficient Numerical Method for Mean Curvature-Based Image Registration Model

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Abstract. Mean curvature-based image registration model firstly proposed by Chumchob-Chen-Brito (2011) offered a better regularizer technique for both smooth and non-smooth deformation fields. However, it is extremely challenging to solve efficiently this model and the existing methods are slow or become efficient only with strong assumptions on the smoothing parameter β. In this paper, we take a different solution approach. Firstly, we discretize the joint energy functional, following an idea of relaxed fixed point is implemented and combine with Gauss-Newton scheme with Armijo's Linear Search for solving the discretized mean curvature model and further to combine with a multilevel method to achieve fast convergence. Numerical experiments not only confirm that our proposed method is efficient and stable, but also it can give more satisfying registration results according to image quality.

AMS subject classifications: 65F10, 65M55, 68U10
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1. Introduction

Image registration which is also called image matching or image warping is one of the most useful and fundamental tasks in imaging processing domain. Its main idea is to find a reasonable spatial geometric transformation between given two images of the same object taken at different times or from different devices or perspectives, such that a transformed version of the first image is similar to the second one as much as possible. It is often encountered in many fields such as astronomy, art, biology, chemistry, medical imaging and remote sensing and so on. For a good overview about these applications, see e.g. [6, 9, 24, 27, 28, 33].

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Usually, a variational image registration model can be described by following form: given two images, one kept unchanged is called reference R and another kept transformed is called template image T. They can be viewed as compactly supported function, $R, T : \Omega \to V \subset \mathbb{R}_0^+$, where $\Omega \subset \mathbb{R}^d$ be a bounded convex domain and d denotes spatial dimension of the given images. The purpose of registration is to look for a transformation φ defined by

$$\varphi : \mathbb{R}^d \to \mathbb{R}^d,$$

such that transformed template image $T_\varphi(\boldsymbol{x}) := T(\varphi(\boldsymbol{x}))$ is similar to R as much as possible. To be more intuitive to understand how a point in the transformed template $T(\varphi(\boldsymbol{x}))$ is moved away from its original position in T, we can split the transformation φ into two parts: the trivial identity part and displacement $\boldsymbol{u}$, $\boldsymbol{u} : \mathbb{R}^d \to \mathbb{R}^d$, $\boldsymbol{u} : \boldsymbol{x} \to \boldsymbol{u}(\boldsymbol{x}) = (u_1(\boldsymbol{x}), u_2(\boldsymbol{x}), \cdots, u_d(\boldsymbol{x}))^\top$, that is to say

$$\varphi(\boldsymbol{x}) = \boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}),$$

thus it is equivalent to find the transformation φ and the displacement $\boldsymbol{u}$. The transformed template image $T(\varphi(\boldsymbol{x})) = T(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}))$ can be denoted $T(\boldsymbol{u})$. In summary, the desired displacement $\boldsymbol{u}$ is a minimizer of the following joint energy functional

$$\min_{\boldsymbol{u}} \left\{ \mathscr{J}_\alpha[\boldsymbol{u}] = \mathscr{D}(\boldsymbol{u}) + \alpha \mathscr{R}(\boldsymbol{u}) \right\}, \tag{1.1}$$

where

$$\mathscr{D}(\boldsymbol{u}) = \frac{1}{2} \int_\Omega \big(T(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - R(\boldsymbol{x}) \big)^2 d\boldsymbol{x} \tag{1.2}$$

represents similarity measure which quantifies distance or similarity of transformed template image $T(\boldsymbol{u})$ and reference R, $\mathscr{R}(\boldsymbol{u})$ is regularizer which rules out unreasonable solutions during registration process, and $\alpha > 0$ is a regularization parameter which balance similarity and regularity of displacement.

And non-surprisingly, different regularizer techniques can produce different registration model, and the choice of regularizer techniques is very crucial for the solution and its properties, more details see [28]. At present, the common regularizer techniques such as diffusion-, elastic-, or linear curvature-based image registration can generate globally smooth displacement, more details see [12, 14, 22, 23, 25, 28, 34] and reference therein. However, these techniques become poor when displacement $\boldsymbol{u}$ is discontinuous. Total variation-based image registration is better for preserving discontinuities of the displacement, see [15, 16, 31]. Nevertheless, the TV model may not give satisfactory registration results for smooth displacement. In this paper, we consider mean curvature regularizer which is able to solve both smooth and non-smooth registration problems as introduced by Chumchob-Chen-Brito [13]:

$$\mathscr{R}^{\mathrm{CCB}}(\boldsymbol{u}) = \frac{1}{2} \sum_{l=1}^{2} \int_\Omega (\kappa(u_l))^2 d\boldsymbol{x}, \tag{1.3}$$