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A Multi-Exposure Variational Method for Retinex

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Abstract. Retinex theory explains how the human visual system perceives colors. The goal of retinex is to decompose the reflectance and the illumination from the given images and thereby compensating for non-uniform lighting. The existing methods for retinex usually use a single image with a fixed exposure to restore the reflectance of the image. In this paper, we propose a variational model for retinex problem by utilizing multi-exposure images of a given scene. The existence and uniqueness of the solutions of the proposed model have been elaborated. An alternating minimization method is constructed to solve the proposed model and its convergence is also demonstrated. The experimental results show that the proposed method is effective for reflectance recovery in retinex problem.

AMS subject classifications: 65K10, 68U10, 91-08

Key words: Retinex, reflectance, illumination, multi-exposure images, alternating minimization algorithm.

1. Introduction

Retinex theory was first proposed by Edwin H. Land in [11] which explains how the human visual system perceives colors. Upon this theory, color sensations correlate with the intrinsic reflectance of objects and are independent of the radiance values captured by eyes. Therefore, human visual system (HVS) can identify the same colors of a given scene under varying illumination conditions, which is commonly regarded as the color constancy, see, for instance, [11–13]. Based on retinex theory, eyes can see colors correctly when light is low, while cameras and video cameras can not manage this well. Images taken under different illumination levels may shift the real color of the object. In retinex theory, it is assumed that the observed image intensity S can be decomposed as pixel-wise product of two components, they are reflectance function R and illumination function L as

$$S = RL. \tag{1.1}$$

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In order to compensate for the non-uniform lighting in a given image and enhance the contrast of images, the primary task of retinex is to find efficient methods to separate the reflectance R from the observed intensity S. The retinex theory is widely applied in image editing [19], shadow removal [5], multi-spectral image fusion [22], image and video fusion for context enhancement [20] and high dynamic range compression [4]. There have been many methods for retinex proposed in the literature. For example, the well-known path-based algorithms which were put forward by Land in [12, 13] and the one put forward by Brainard et al. in [2]. Path-based algorithms require to tune many parameters and the implementation is very complicate. A recursive matrix calculation was designed to replace the path computation and recursive algorithms were proposed in [6, 7]. The single scale retinex model was proposed in [8], though the optimized single scale retinex result is short of human observation, it succeeds in producing the correct beige scene color and some dynamic range compression of the shadow. The multi-scale retinex was proposed in [9] which more closely approaches the performance of human vision. In 2009, Bertalmo, Caselles, and Provenzi [1] proposed a kernel-based retinex method in which the main computation is to get the expectation value of a suitable random variable weighted by a kernel function. The partial differential equation based algorithms are important methods for retinex. For example, the methods proposed by Morel et al. in [16, 17] utilize fast Fourier transformation to perform the computation cheaply to get the decomposition of reflectance and illumination in recorded images. Morel et al. also further demonstrated that the random walk method and the partial differential equation formulation are equivalent. Efficient variational methods for retinex have surged in recent years. Ma et al. [14] proposed a model in which the L_1 regularization is used to recover sharp edges and boundaries of the reflectance component and a fast approach based on Bregman iteration is designed to solve the model. Kimmel et al. [10] presented a variational method based on H_1-norm regularization for the reflectance function. Ma and Osher established a total variation and nonlocal TV regularized model in [15]. In [18], Ng and Wang proposed a model for retinex which consists of a data-fidelity term, a total variation term for reflectance function, an H_1-norm regularization term for illumination function. An alternating minimization method is designed to solve this problem. In this method, due to the blurring recovering effect of the recovered R from the model, $R' = S/L$ instead of R is used in image enhancement and illumination compensation. In 2014, Zosso et al. in [26] and Wang, Ng in [24] constructed methods in which the nonlocal total variation regularization of the reflectance function is used in order to improve the reflectance recovering effect. Wang and He [25] proposed a variational model which has the same data-fidelity term as the one used in [18], but two barrier functions are added. The details and edges of the recovered reflectance R from this model is clearer and sharper than the one got from [18]. In [3], Chang et al. used sparse and redundant representations of the reflectance component in the retinex model over a learned dictionary and more details are revealed in the low-light part.

In the above mentioned methods, only single image with a fixed exposure is used to restore the reflectance of the image. Although reflectance is a constant property and related to the physical characteristics of the material object, in practice, certain parts of image details may be lost in the saturated or over-dark regions due to the different exposure