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Wavelet Based Restoration of Images with Missing or Damaged Pixels

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Abstract. This paper addresses the problem of how to restore degraded images where the pixels have been partly lost during transmission or damaged by impulsive noise. A wide range of image restoration tasks is covered in the mathematical model considered in this paper – *e*.g. image deblurring, image inpainting and super-resolution imaging. Based on the assumption that natural images are likely to have a sparse representation in a wavelet tight frame domain, we propose a regularization-based approach to recover degraded images, by enforcing the analysis-based sparsity prior of images in a tight frame domain. The resulting minimization problem can be solved efficiently by the split Bregman method. Numerical experiments on various image restoration tasks – simultaneously image deblurring and inpainting, super-resolution imaging and image deblurring under impulsive noise – demonstrated the effectiveness of our proposed algorithm. It proved robust to mis-detection errors of missing or damaged pixels, and compared favorably to existing algorithms.

Key words: Image restoration, impulsive noise, tight frame, sparse approximation, split Bregman method.

1. Introduction

A digital image may be distorted or degraded during image formation or transmission, which can extend to an unacceptable loss of visual image quality. How to recover degraded images has long been a fundamental problem in image processing. There exist many types of image degradations in practice – *e*.g. image blurring by out-of-focusing or camera shake during image acquisition, image/film deterioration due to dust spots or cracks in film, low resolution of images due to physical limits of digital cameras, and noisy images caused by

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noisy sensors or transmission errors. For simplicity, if we denote images as vectors in $\mathbb{R}^n$ by concatenating their columns, the observed degraded version of the latent image u can usually be modeled as

$$f = Hu + \epsilon, \tag{1.1}$$

where f is the observed degraded image, u is the latent image, ϵ is the image noise and the matrix H denotes the degrading operator. The image restoration task is then to reverse the effect of the operator H on f to recover the latent image u. It is well known that image restoration is an ill-conditioned inverse problem sensitive to image noise. It makes things even harder when the complete version of the degraded image f is unavailable. This can happen when, for example, some image pixels go missing during the transmission, or some image regions are damaged due to scratches in films. In all these cases, only a subset of image pixels is available or reliable. If Λ denotes the index set of all available image pixels, then the image degradation model (1.1) becomes

$$P_\Lambda f = P_\Lambda(Hu + \epsilon) \tag{1.2}$$

where P_Λ is the projection operator, defined by a diagonal matrix with diagonal entries 1 for the indices in Λ and 0 otherwise.

The goal of this paper is to develop a robust algorithm to solve (1.2) – i.e. to restore the latent image u from its incomplete, degraded and noisy version $P_\Lambda f$. The quite generic operator H in (1.2) includes many types of image degradations. For example, H may be the matrix form of the discrete convolution operator for image deblurring, a projection matrix for image inpainting, or an identity matrix for image denoising. Image restoration with missing or damaged pixels is not only ill-conditioned but also ill-posed with an infinite number of solutions. To recover the latent image u, we need to assume that there exists some structure prior or image redundancy such that all pixels can be inferred from partial degraded information for u. There have been many image priors proposed in the past – *e*.g. the Tikhonov functional-based smoothness prior for images [41], total variation functional [20, 38] or Mumford-Shah functional-based piece-wise smoothness prior [33] for cartoon images, and the exemplar-based local patch redundancy prior of natural images [23]. In recent years, sparsity-based priors of images in certain domains have been used widely in many image restoration tasks, based on the observation that images usually have sparse representations (or sparse approximations) in some transformed domains. For example, there can be Fourier or windowed Fourier transforms, local cosine transforms, wavelet or framelet transforms, or discrete gradient operators. In particular, the sparsity prior of images in tight frame systems [24, 37] has been used successfully in many image restoration tasks, such as image inpainting [11, 12, 16], non-blind image deblurring [15, 16] and blind motion deblurring [13, 14]. All of this sparsity-based research motivates us to investigate the application of the sparsity prior of images in a tight frame domain to solve (1.2), for image restoration with missing or damaged pixels.

In order do so, one may find a sparse solution of (1.2) in the tight frame domain, which can be approximated by solving an ℓ_1-norm regularized minimization problem. Based on different regularization strategies, there are three types of sparsity priors – viz. synthesis-based sparsity prior [15], balanced sparsity prior [12], and analysis-based sparsity prior