East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.090312.190412a

Vol. **2**, No. 2, pp. 150-169
May 2012

Coefficient of Variation Based Image Selective Segmentation Model Using Active Contours

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Received 9 March 2012; Accepted (in revised version) 19 April 2012

Available online 27 April 2012

Abstract. Most image segmentation techniques efficiently segment images with prominent edges, but are less efficient for some images with low frequencies and overlapping regions of homogeneous intensities. A recently proposed selective segmentation model often works well, but not for such challenging images. In this paper, we introduce a new model using the coefficient of variation as a fidelity term, and our test results show it performs much better in these challenging cases.

AMS subject classifications: 68U10, 62G30

Key words: Segmentation, Coefficient of Variation (CoV), level set, functional minimisiation, Total Variation (TV).

1. Introduction

In image segmentation, the main issue is to extract features according to a given criterion [3, 5, 6, 10, 11, 13, 15]. There are two important method categories. The first category refers to edge-based methods, where active contours have proven their effectiveness [5, 8, 13, 20]. The general idea behind an active contours model is to apply partial differential equations (PDEs) to deform a curve towards the boundaries of the object of interest, so the contour is driven towards image edges. For edge detection, most models use an edge detector function which depends on the gradient of a given image [3, 10, 11]. The second category contains region-based methods, including active contour models involving minimum description length criteria [14], region growing and emerging [2], Mumford-Shah functional minimisation [18] and watershed algorithms [24] as examples.

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Let $z(x,y)$ be a given image defined on a rectangular domain Ω. Mumford and Shah (MS) [18] proposed the general model

$$\min_{u,\Gamma} F(u,\Gamma) = \mu\,.\,length(\Gamma) + \lambda \int_{\Omega} |z-u|^2 dx + \int_{\Omega\backslash\Gamma} |\nabla u|^2 dx$$

to automatically find the edge Γ of z by a piecewise smooth function u. The Chan-Vese (CV) [6] model is a special case of the piecewise constant MS model when restricted to only two phases. Since the CV model is not based on the gradient of the image $z(x,y)$ for the stopping process, it can detect contours both with and without gradients. The CV active contour model uses the energy minimisation functional given by:

$$F(c_1,c_2,\Gamma) = \mu.length(\Gamma) + \lambda_1 \int_{inside(\Gamma)} |z-c_1|^2\, dx\, dy + \lambda_2 \int_{outside(\Gamma)} |z-c_2|^2\, dx\, dy,$$

where z is a given image, Γ is an unknown boundary, and c_1 and c_2 are constants that depend on Γ and represent the average value of z inside and outside of Γ respectively.

The above categories of segmentation models are global, because all global features are to be segmented. Although useful, in certain segmentation problems we need to segment a particular object and not all objects. Selective segmentation is a task in which an object or region of interest is detected, given additional information of geometric constraints in the form of list of points near the object or region.

Based on the work of Refs. [6, 10, 11], we recently proposed a mixed model of edge-based and region-based methods that provved more robust for noisy images [3]. However, this model can produce spurious objects — i.e. fails the selection in some cases. Now we equip our model with a new type of fidelity term for it to perform better, even when edges are not prominent or an image has overlapping regions with almost homogeneous intensities. The fidelity term is based on a coefficient of variation, and our experimental results demonstrate the superior performance of this new model.

This paper is organised in the following way. A review of the previous model [3] is presented in Section 2. Our new model of minimisation and the Euler-Lagrange equation are discussed in Section 3. We describe a semi-implicit method and an additive operator splitting (AOS) method for solving the PDE on Section 4, and give some experimental results in Section 5.

2. BC Model

To segment a given image z or find the boundary Γ of a desirable feature, the recent Badshah and Chen (BC) model [3] solves

$$\min_{c_1,c_2,\Gamma} F(\Gamma, c_1, c_2)\,,$$