East Asian Journal on Applied Mathematics Vol. **7**, No. 2, pp. 343-362
doi: 10.4208/eajam.100316.270117a May 2017

Uncertainty Quantification of Derivative Instruments

Xianming Sun[1,2,*] and Michèle Vanmaele[2]

[1] *School of Finance, Zhongnan University of Economics and Law, Wuhan 430073, China.*
[2] *Department of Applied Mathematics, Computer Science and Statistics, Ghent University, Ghent B-9000, Belgium.*

Received 10 March 2016; Accepted (in revised version) 27 January 2017.

Abstract. Model and parameter uncertainties are common whenever some parametric model is selected to value a derivative instrument. Combining the Monte Carlo method with the Smolyak interpolation algorithm, we propose an accurate efficient numerical procedure to quantify the uncertainty embedded in complex derivatives. Except for the value function being sufficiently smooth with respect to the model parameters, there are no requirements on the payoff or candidate models. Numerical tests carried out quantify the uncertainty of Bermudan put options and down-and-out put options under the Heston model, with each model parameter specified in an interval.

AMS subject classifications: 62P05, 65C05, 65C50, 68U20

Key words: Parameter uncertainty, Derivative pricing, Smolyak algorithm, Monte Carlo, Entropy

1. Introduction

In the context of pricing and hedging exotic over-the-counter (OTC) derivatives, the price process of the underlying asset can be described by different types of parametric models, such as stochastic volatility models (e.g. Ref. [26]) and fraction Brownian motion models (e.g. Ref. [33]). The parameters of these models can be estimated by model calibration, but there are two issues: (1) different kinds of models can be perfectly calibrated to the same market data [38]; and (2) different calibration methods may yield different estimations for each parameter in a specific model [21].

These empirical issues, together with limited knowledge of the market dynamics, confront an agent with ambiguity about which model is the best to value a target derivative, especially since the calibrated models can provide quite different values for the target derivative. Indeed, model and parameter ambiguities are ubiquitous whenever a parametric model is employed, in two different categories according to Ref. [30] — viz. risk and uncertainty. Model (parameter) risk relates to those settings where the probabilities

*Corresponding author. *Email address:* `Xianming.Sun@hotmail.com` (X. Sun), `Michele.Vanmaele@ugent.be` (M. Vanmaele)

of the candidate models (parameter realisations) are known, while model (parameter) uncertainty arises from a lack of knowledge of the probabilistic information on these models (parameter realisations). Here we focus on numerically quantifying model (parameter) uncertainty embedded in a derivative instrument

Model uncertainty is a growing concern for regulation and risk management [6,17], and researchers have proposed both model-independent and model-dependent approaches to deal with model uncertainty in applications. A model-independent approach for the value bounds of a derivative is an effective way to eliminate model (parameter) uncertainty, by providing an agent with a conservative reference to quote a target derivative — see e.g. Refs. [5, 9, 12, 27, 40, 41]. However, these model-independent approaches may not work when the target derivative is too complex to be hedged model-independently, and a robust alternative is to employ a model-dependent approach by considering a set of plausible models to value the derivative. At the parameter level, a confidence interval rather than the point estimation of each model parameter can be employed, to account for parameter uncertainty in the setting of option pricing. We believe this idea was first proposed in Refs. [2] and [32], and traders and institutions often attack model (parameter) uncertainty through a worst-case approach — e.g. by stress testing portfolios [37]. Following a worst-case approach, Cont [11] proposed an uncertainty measure — viz. the spread between lower and upper bounds of the target derivative value when a set of plausible models are employed, which accounts for both hedgeable and unhedgeable risk.

However, although a set of meaningful axioms is satisfied, to date little attention has been paid on how to calculate this uncertainty measure accurately for exotic derivatives under complex parametric models — but here we propose an accurate and efficient numerical method to calculate the worst-case value of a derivative instrument under model and parameter uncertainties, combining the Smolyak sparse-grid interpolation approximation with the Monte Carlo-based optimisation method. The only assumption we make is that the derivative value function is smooth enough with respect to the model parameters, which should be a premise feature whenever a parametric model is employed to value an exotic derivative, for otherwise the model user suffers an additional risk if the model parameters are not estimated with high accuracy. Our second objective is to use entropy to complement the uncertainty measure proposed by Cont [11] to assess the uncertainty of the target derivative value with respect to model and parameter uncertainties. In some cases, his uncertainty measure may have the same value spread (not value bounds) for two different derivatives, when cannot be used to distinguish the uncertainty embedded in them. The entropy measure can be used as an alternative, to further quantify their uncertainty from the perspective of the amount of information provided by the ensemble of derivative values.

Our work reflects two strands in the literature. The first and foremost is the calculation of robust values of complex exotic derivatives in a model-dependent framework, where the model parameters are specified in terms of an interval rather than a point in each parameter direction — e.g. the famous volatility uncertainty model [2,32], where the underlying asset price follows

$$dS_t = rS_t dt + \sigma_t S_t dW_t, \quad \sigma_t \in [\sigma_{min}, \sigma_{max}].$$