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Modulus-based Synchronous Multisplitting Iteration Methods for an Implicit Complementarity Problem

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Abstract. We construct modulus-based synchronous multisplitting iteration methods to solve a large implicit complementarity problem on parallel multiprocessor systems, and prove their convergence. Numerical results confirm our theoretical analysis and show that these new methods are efficient.

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Key words: Implicit complementarity problem, synchronous, iterative method, modulus-based multisplitting.

1. Introduction

Implicit complementarity problems arise in a variety of applications in economics and engineering such as the Nash equilibrium point of a bimatrix game, contact problems, and the free boundary problem for journal bearings — cf. Ref. [9,13,24] and references therein. The modulus method is a classical iteration procedure to solve linear complementarity problems [12, 18, 26], Hadjidimos & Tzoumas [15, 16] proposed extrapolated modulus algorithms, and Bai [3] presented the modulus-based matrix splitting iteration method.

Given the computational requirements of modern high-speed multiprocessor environments, multisplitting iterative methods are powerful tools for large sparse linear complementarity problems [1, 2, 4–7, 11, 19, 23, 25]. Li & Zeng [20, 21] considered several synchronous and asynchronous multisplitting iteration schemes for solving a class of nonlinear complementarity problem when the system matrix is an H-matrix. Recently, via an equivalent reformulation of the linear complementarity problem as a system of fixed-point equations, Bai & Zhang [7] constructed modulus-based synchronous multisplitting (MSM) iteration methods that are suitable for implementation on multiprocessor systems. As one

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only needs to solve subsystems of linear equations at each iteration step in these modulus-based synchronous multisplitting (MSM) iteration methods, they are better than the general multisplitting iteration methods since they produce linear complementarity subproblems [8, 10, 15, 22, 27–32]. We extend these methods to solve an implicit complementarity problem, noting that it includes a corresponding linear complementarity problem.

Consider the following implicit complementarity problem that we denote as ICP(q, M):

Find a pair of real vectors w and $u \in R^n$ such that

$$u - m(u) \geq 0, \quad w := Mu + q \geq 0, \quad (u - m(u))^T (Mu + q) = 0 \tag{1.1}$$

where $M \in R^{n\times n}$ is a given sparse matrix and $q \in R^n$ is a real vector, and $m(\cdot)$ is an invertible mapping from R^n into itself.

In this article, we extend the MSM iteration methods to solve the $ICP(q, M)$. In Section 2, we present some necessary notation, definitions and lemmas. In Section 3, our MSM methods for solving the $ICP(q, M)$ are established. We prove the convergence of the MSM iteration methods when the system matrix is an H_+-matrix in Section 4. Numerical results presented in Section 5 show that the MSM iteration methods are computationally more efficient than modulus-based splitting iteration methods.

2. Preliminaries

Suppose that matrices $A = (a_{ij})$, $B = (b_{ij}) \in R^{n\times n}$. We shall write $A \geq B$ $(A > B)$ if $m_{ij} \geq n_{ij}$ $(m_{ij} > n_{ij})$ holds for all $1 \leq i \leq n$, $1 \leq j \leq n$. If O is a null matrix and $A \geq O$, A is called a nonnegative matrix. and $|A|$ denotes the nonnegative matrix with entries $|a_{ij}|$. If $A \in R^{n\times n}$ is a real $n \times n$ matrix, its comparison matrix is $\langle A \rangle = (\langle a_{ij} \rangle) \in R^{n\times n}$ with

$$\langle a \rangle_{ij} = \begin{cases} |a_{ij}|, & i = j \\ -|a_{ij}|, & i \neq j \end{cases}, \quad i, j = 1, 2, \cdots, n .$$

The matrix M is called an M-matrix if its off-diagonal entries are all non-positive and $M^{-1} \geq O$; M is called an H-matrix if its comparison matrix $\langle M \rangle$ is an M-matrix; an H-matrix M is called an H^+-matrix if it has a positive diagonal matrix; and if M is an M-matrix and Ω is a positive diagonal matrix, then $M \leq B \leq \Omega$ implies that B is an M-matrix. If F is a non-singular matrixm then $M = F - G$ is called a splitting of the matrix; and if $\langle M \rangle = \langle F \rangle - |G|$, then $M = F - G$ is called an H-compatible splitting.

Lemma 2.1 ([14]). *Let $A \in R^{n\times n}$ be an H - matrix, $D = diag(A)$ and $B = D - A$. Then*

(1) matrix A is non-singular;
(2) $|A^{-1}| \leq \langle A \rangle^{-1}$; and
(3) $|D|$ is non-singular and $\rho(|D|^{-1} \cdot |B|) < 1$.

Lemma 2.2 ([17]). *Let $M = F - G$ be a splitting of the matrix $M \in R^{n\times n}$, r a positive constant, and Ω a positive diagonal matrix. For the problem (1.1), we then have the following:*