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Exact Traveling Wave Solutions of a Fractional Sawada-Kotera Equation

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Abstract. Exact traveling wave solutions of the fifth order space-time fractional Sawada-Kotera equation are derived by generalised $\exp(-\Phi(\xi))$-expansion and an improved fractional sub-equation method. Among the solutions obtained there are hyperbolic, trigonometric, exponential and rational ones. The methods are simple, efficient and can be applied to other nonlinear problems.

AMS subject classifications: 35N05,35C07,35C08,35Q53

Key words: Fractional Sawada-Kotera equation, exact solutions.

1. Introduction

Nonlinear complex phenomena play an important role in physical sciences, and the generalised KdV equation is widely used in the description of waves in nonlinear LC circuits, shallow and stratified internal waves, ion-acoustic waves [1–3]. The higher-order members of the KdV hierarchy find applications in internal and surface waves, gravity-capillary waves, floating ice coat and many other fields — cf. Refs. [3–8]. Exact solutions of the corresponding nonlinear PDEs can provide valuable information about natural phenomena. Considerable efforts spent on the solution of such equations resulted in new methods — e.g. the Hirota bilinear scheme [9], inverse scattering scheme [1], Bäcklund transform method [4], homogeneous balance scheme [10], generalised $\exp(-\Phi(\xi))$-expansion method [11], (G'/G)-expansion method [12] and so on [13–15].

Recently, special attention has been paid to analytic solutions of differential equations [16–27]. It is often assumed that the solution is a polynomial of degree determined by the homogeneous balance principle [42]. Let us also note the sub-equation and expansion methods, which allowed to derive a number of analytical solutions to linear and non-linear PDEs, including exponential, polynomial and rational solutions, trigonometric wave solutions, solitary wave solutions in hyperbolic form and many others — cf. Refs. [16–22].

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On the other hand, in the last three decades, fractional differential equations (FDEs) found numerous applications, especially in fluid flows, viscoelasticity, control theory, chemical physics, electrical networks [28–30]. Consequently, the methods for their solutions have been developed. Thus using the Mittag-Leffler function in exp-function scheme, Zhang [32] obtained analytical results for FDEs [31]. This method has been then employed to find periodic, solitary wave and compact like solutions for Klein-Gordon equation [33], Maccari's system [34], Broer Kaup system [35], combined KdV and mKdV equation [36], Toda lattice equation [37]. It was noted that choosing special parameters in exp-function approach, one can derive better solutions than those obtained by the existing schemes. Using results of [32], S. Zhang and H. Zhang [38] proposed the fractional sub-equation method. Based on the modified Riemann-Liouville derivatives [39–41], the homogeneous balance principle [42] and symbolic calculation, this approach allows to find analytic solutions of non-linear fractional problems. In the present work, we utilise the generalised $\exp(-\Phi(\xi))$-expansion method [14, 46] and an improved fractional sub-equation scheme [47, 48] to derive the exact traveling wave solution of the space-time fractional equation [5,43] of the fifth order:

$$D_t^{\alpha}u + 45u^2 D_x^{\beta}u + 15D_x^{\beta}uD_x^{2\beta}u + 15uD_x^{3\beta}u + D_x^{5\beta}u = 0, \quad 0 < \alpha, \beta \leq 1. \tag{1.1}$$

For $\alpha = 1, \beta = 1$ this is the popular unidirectional nonlinear Sawada-Kotera equation studied in Refs. [4, 7, 44, 45]. However, to the best of our knowledge, the above mentioned methods have not been applied to the Eq. (1.1). On the other hand, the traveling wave solutions of this equation have been obtained by (G'/G)-expansion method [49], tanh-sech method [50] and Chebyshev wavelet method [51].

In Section 2, definitions related to the modified Riemann-Liouville derivative are given. Some details of the improved fractional sub-equation technique and generalised $\exp(-\Phi(\xi))$-expansion method are presented in Section 3. Section 4 deals with exact traveling wave solutions of the Eq. (1.1). Our conclusions are in Section 5.

2. Preliminaries

Let us recall the Jumarie modification of the Riemann-Liouville derivative

$$g^{(\alpha)}(x) := \lim_{h\to 0}\left(\frac{\sum_{n=0}^{\infty}(-1)^n \begin{pmatrix} \alpha \\ n \end{pmatrix} g[x+(\alpha-n)h]}{h^{\alpha}}\right), \tag{2.1}$$

of order α. According to Ref. [38], derivative (2.1) can be represented in the form

$$D_x^{\alpha}g(x) = \begin{cases} \frac{1}{\Gamma(-\alpha)}\frac{d}{dx}\int_0^x (x-\xi)^{-\alpha-1}[g(\xi)-g(0)]d\xi, & \text{if } \alpha < 0, \\ \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_0^x (x-\xi)^{-\alpha}[g(\xi)-g(0)]d\xi, & \text{if } 0 < \alpha < 1, \\ [g^{(\alpha-n)}(x)]^{(n)}, & \text{if } n \leq \alpha < n+1, n \geq 1. \end{cases}$$