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Abundant Mixed Lump-Soliton Solutions to the BKP Equation

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Abstract. Applying Maple symbolic computations, we derive eight sets of mixed lump-soliton solutions to the $(2+1)$-dimensional BKP equation. The solutions are analytic and allow the separation of lumps and line solitons.

AMS subject classifications: 35Q51, 35Q53, 37K40

Key words: Lump solution, soliton solution, symbolic computation.

1. Introduction

It is well known that solitons describe various significant nonlinear phenomena in nature [1, 39] and the Hirota bilinear method provides a power tool for solving integrable equations [16]. Positons and complexitons are other typical solutions of integrable equations [25, 43], and the interaction between different classes of solutions leads to a better understanding of nonlinear phenomena [35]. In particular, the long wave limits of solitons generate lump solutions, rationally localized solutions in all directions in space, and many

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other ones — cf. Refs. [1,40]. Hirota bilinear forms play a crucial role in finding such exact solutions but the algorithm heavily relies on try and error experiments [5,16].

Let us recall [30] that the KP equation

$$(u_t + 6uu_x + u_{xxx})_x - u_{yy} = 0,$$

has the following set of lump solutions:

$$\begin{aligned} u = 2(\ln f)_{xx},\ f &= \Big(a_1 x + a_2 y + \frac{a_1 {a_2}^2 - a_1 {a_6}^2 + 2\, a_2 a_5 a_6}{{a_1}^2 + {a_5}^2} t + a_4\Big)^2 \\ &+ \Big(a_5 x + a_6 y + \frac{2 a_1 a_2 a_6 - {a_2}^2 a_5 + a_5 {a_6}^2}{{a_1}^2 + {a_5}^2} t + a_8\Big)^2 + \frac{3({a_1}^2 + {a_5}^2)^3}{(a_1 a_6 - a_2 a_5)^2}, \end{aligned}$$

where a_i are arbitrary parameters such that $a_1 a_6 - a_2 a_5 \neq 0$. This set contains a subset of lump solutions of the form

$$u = 4 \frac{-[x + ay + (a^2 - b^2)t]^2 + b^2(y + 2at)^2 + 3/b^2}{\{[x + ay + (a^2 - b^2)t]^2 + b^2(y + 2at)^2 + 3/b^2\}^2}, \tag{1.1}$$

with two free parameters a and b [38]. The situation is not unique and there are many integrable equations with lump solutions — e.g. three-dimensional three-wave resonant interactions [21], the BKP equation [14,45], the Davey-Stewartson equation II [40], the Ishimori-I equation [17] and so on. Besides, non-integrable equations, such as $(2+1)$-dimensional generalized KP, BKP and Sawada-Kotera equations, also have lump solutions — cf. Refs. [8,32,36,48,53].

It is worth noting that the general rational solutions of integrable equations have been derived within the framework of Wronskian, Casoratian, Grammian and Pfaffian formulations [1,16]. The set of equations studied contains a variety of physically significant equations such as the KdV and Boussinesq equations, the nonlinear Schrödinger equation in $(1+1)$-dimensions, the KP and BKP equations in $(2+1)$-dimensions, and the Toda lattice equation in $(0+1)$-dimensions [2,7,13,34,35]. General rational solutions of nonlinear partial differential equations — e.g. generalized bilinear differential equations have been also discussed [3,37,47,50–52].

Here we consider a $(2+1)$-dimensional BKP equation of [9,18] — viz.

$$P_{BKP}(u) := (u_t + 15uu_{xxx} + 15{u_x}^3 - 15u_x u_y + u_{5x})_x - 5u_{xxxy} - 5u_{yy} = 0. \tag{1.2}$$

This is a first member in the BKP integrable hierarchy [6,41], represented by the $(2+1)$-dimensional generalization of the Caudrey-Dodd-Gibbon-Sawada-Kotera equation

$$v_t + 15vv_{xxx} + 15v_x v_{xx} + 45v^2 v_x + v_{5x} = 0. \tag{1.3}$$

If $v = u_x$ and the function u depends on x and t only, then (1.2) becomes the Eq. (1.3). The underlying spectral problem

$$-\phi_y + \phi_{xxx} + (3v - \lambda)\phi = 0, \tag{1.4}$$