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An a Posteriori Error Estimator for a Non-Conforming Domain Decomposition Method for a Harmonic Elastodynamics Equation

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Abstract. We develop a reliable residual-based a posteriori error estimator for a non-conforming method with non-matching meshes for a harmonic elastodynamics equation and show that the approximation method converges with an optimal order to the exact solution. Moreover, we propose an adaptive strategy to reduce computational cost and derive better approximations for problems with singularities and with large approximating systems. Numerical experiments confirm theoretical conclusions.

AMS subject classifications: 65N30, 65N55

Key words: Harmonic elastodinamics equation, domain decomposition method, Nitsche method, non-matching mesh, a posteriori error estimator, adaptive method.

1. Introduction

The elastodynamics equation arises in the vibration of beams supporting the buildings and bridges and in seismology while considering the earth as a medium where waves spread. In this paper, we apply the domain decomposition methods to the harmonic elastodynamics equation. The original domain is split into subdomains and the problem is solved separately in every subdomain that is suitable for parallel computing — cf. Ref. [16]. For the subdomains having overlapping areas only on the interface, we employ a non-overlapping domain decomposition method and conforming or non-conforming technique to impose continuous solutions on the interface. On the other hand, the non-matching meshes near the interface of the subdomains lead to non-conforming methods. We recall that mortar,

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Nitsche, standard Schwarz methods and a number of methods based on domain decomposition can produce continuous solutions on the interface. The mortar scheme introduces weak interface equations via Lagrange multipliers, whereas the Nitsche method [12] controls the discontinuity by adding a penalty term in the discrete formulation. The non-matching meshes appear, if the subdomain meshes are generated independently or if some of them are not refined. The use of different meshes is of special interest in the case of corner singularities or different material parameters.

There is a vast literature on implementation of domain decomposition using direct procedures. For the Poisson equation, Becker *et al.* [3] presented a Nitsche method and a posteriori error estimates for non-matching meshes and Hansbo *et al.* [10] introduced a residual-type estimator. Becker [2] presented an adaptive strategy in Nitsche scheme for the Navier–Stokes equation, Heinrich and Nicaise [11] applied a Nitsche-type mortar method to transmission problems, Fritz *et al.* [9] used mortar and Nitsche techniques in elasticity problem, Vergara [15] employed a Nitsche scheme in fluid dynamics, Boiveau and Burman [4] analyzed the stability of a Nitsche scheme in linear elasticity. There is extensive literature on discrete non-conforming schemes with positive definite bilinear forms but to the best of authors' knowledge, the non-positive definite problems have not been studied yet.

The main goal of this work is to extend the Nitsche method [3,9] on a two-dimensional harmonic elastodynamics equation, the discrete bilinear form of which is, generally, not positive definite but satisfies the Gårding inequality, so it can be treated as a compact perturbation. We consider two non-overlapping domain decompositions and non-matching triangulations on the interface of the subdomains. Moreover, we present a novel a posteriori residual error estimator for the associated discrete weak formulation and develop an adaptive strategy to improve the convergence of the method and reduce computational cost, especially for problems with singularities. We also establish the stability and convergence of a non-conforming method. However, the numerical scheme we use, is still affected by locking phenomena for $\mathbb{P}_1$ finite elements in pure displacement formulation with Poisson's ratio close to 0.5. The residual a posteriori error estimator is derived for L^2-norm by standard techniques for finite element methods (FEMs). In the proof of its reliability and efficiency, a saturation condition, Aubin–Nitsche trick and arguments of [8, 13] are used. The error estimate allows us to define local indicators and present adaptive algorithms for mesh refinement in non-conforming procedures.

The paper is organized as follows. In Sections 2 and 3 we introduce a boundary value problem and the Nitsche scheme, describe a non-matching discrete weak formulation and its properties. Section 4 contains a reliable residual a posteriori error estimator of a non-conforming discrete problem, the proof of the reliability for a broken discrete $\boldsymbol{H}^1$-norm, and an adaptive strategy for the convergence improvement in the case of singularities. Numerical experiments considered in Section 5, confirm the theoretical results concerning the convergence of the method and the reliability and efficiency of a posteriori error estimator. Our conclusions are in Section 6. In Appendix, we show the efficiency of the error estimator obtained.