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Fluxon Centering in Josephson Junctions with Exponentially Varying Width

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Abstract. Nonlinear eigenvalue problems for fluxons in long Josephson junctions with exponentially varying width are treated. Appropriate algorithms are created and realized numerically. The results obtained concern the stability of the fluxons, the centering both magnetic field and current for the magnetic flux quanta in the Josephson junction as well as the ascertaining of the impact of the geometric and physical parameters on these quantities. Each static solution of the nonlinear boundary-value problem is identified as stable or unstable in dependence on the eigenvalues of associated Sturm-Liouville problem. The above compound problem is linearized and solved by using of the reliable Continuous analogue of Newton method.

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Key words: Fluxon, stability, bifurcation, critical curve, centering magnetic field, centering current.

1. Introduction

In 1962 Josephson observed a supercurrent (i.e. a current that may flow for an indefinitely long time without any applied voltage) across a device subsequently called a Josephson junction (JJ), consisting of two superconductors coupled by a weak link. This discovery, now known as the Josephson effect, led to many numerical and empirical investigations of one-stacked or multi-stacked homogeneous and inhomogeneous JJs [2, 9–11, 16]. There are several ways to model the shape and the influence of the inhomogeneity — e.g. by a Dirac delta function, some finite piece-wise function, or some smooth function. The JJs with variable geometry, typically modelled by smooth functions, have recently received considerable attention [3, 5, 8, 13] and are the subject of this article.

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A basic feature of any JJ is the critical (bifurcation) dependence of the "current-magnetic field", directly related to the stability of the magnetic flux quanta. The respective boundary and eigenvalue problems are quite difficult to solve analytically, even in the one-dimensional case, and this has led to the development and implementation of appropriate numerical procedures. Actually, the stable solutions that are sought that describe so-called fluxons turn out to be admissible space-time distributions in the JJ. However, there are difficulties due to the presence of physical and geometrical parameters, which can significantly affect the solution behaviour. Thus when critical modes of the distributions are considered, the corresponding parameter set should be regarded as a parametric eigenspace that complicates the investigation

Our interest in these modes has also been fostered by the empirical investigations conducted by Benabdalah *et al.* [3]. They found a threshold for the current, beyond which the static distribution becomes unstable with fluxons moving to the narrower end of the device. Their observations lead us to conjecture that the outer magnetic field and current influence and control the location and stability of the Josephson vortices in the JJ. With this in mind, we consider the fluxons in inhomogeneous JJs with exponentially varying width — in particular, the stability of the quant-magnetic field, the centering of the magnetic field and the current, and the relationships and dependencies between them and quantities in the multiparametric space. In Section 2, we pose boundary-value problems for in-line and overlap geometries, and appropriate eigenvalue problems. The linearisation, discretisation and the numerical procedure adopted are considered in Section 3. Our main results, their interpretation, and a final discussion are then presented Sections 4 and 5.

2. Formulation of the Problem

A sine-Gordon wave equation is assumed to govern the space-time evolution of the magnetic flux in the JJ with exponentially varying width [3]. For static distributions of the magnetic flux in JJs with in-line and overlap geometries, we consider two different two-point boundary-value problems (BVP) — viz.

$$\begin{aligned} &-\varphi_{xx} + \sigma\varphi_x + \sin\varphi - \sigma h_e = 0\,, \\ &\varphi_x(0) - h_e + l\gamma = 0\,, \qquad \varphi_x(l) - h_e = 0\,; \end{aligned} \tag{2.1}$$

$$\begin{aligned} &-\varphi_{xx} + \sigma\varphi_x + \sin\varphi - \sigma h_e - \gamma = 0\,, \\ &\varphi_x(0) - h_e = 0\,, \qquad \varphi_x(l) - h_e = 0\,. \end{aligned} \tag{2.2}$$

Here l is the the length of the JJ; and φ_x denoting the normed magnetic field in the junction, h_e the intensity of the outer (bias) magnetic field, γ the density of the normed outer current, and σ the shape parameter, are all dimensionless quantities. [We suppose the width of the JJ varies according the law $W(x) = W_0 \exp(-\sigma x)$.]

In solving the BVP (2.1) or (2.2), we need information about the stability of the sought solution. Moreover, we are interested in the minimal stable state, especially the critical "current-magnetic field" relationship [13]. Given the nonlinearity (2.1) and (2.2) , we