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Comparison of Spectral and Wavelet Estimators of Transfer Function for Linear Systems

M. A. A. Bakar[1,2,*], D. A. Green[2] and A. V. Metcalfe[2]

[1] *School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, UKM Bangi, Selangor, Malaysia.*
[2] *School of Mathematical Sciences, University of Adelaide, South Australia, Australia.*

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Abstract. We compare spectral and wavelet estimators of the response amplitude operator (RAO) of a linear system, with various input signals and added noise scenarios. The comparison is based on a model of a heaving buoy wave energy device (HBWED), which oscillates vertically as a single mode of vibration linear system. HBWEDs and other single degree of freedom wave energy devices such as oscillating wave surge convertors (OWSC) are currently deployed in the ocean, making such devices important systems to both model and analyse in some detail. The results of the comparison relate to any linear system. It was found that the wavelet estimator of the RAO offers no advantage over the spectral estimators if both input and response time series data are noise free and long time series are available. If there is noise on only the response time series, only the wavelet estimator or the spectral estimator that uses the cross-spectrum of the input and response signals in the numerator should be used. For the case of noise on only the input time series, only the spectral estimator that uses the cross-spectrum in the denominator gives a sensible estimate of the RAO. If both the input and response signals are corrupted with noise, a modification to both the input and response spectrum estimates can provide a good estimator of the RAO. A combination of wavelet and spectral methods is introduced as an alternative RAO estimator. The conclusions apply for autoregressive emulators of sea surface elevation, impulse, and pseudorandom binary sequences (PRBS) inputs. However, a wavelet estimator is needed in the special case of a chirp input where the signal has a continuously varying frequency.

AMS subject classifications: 65M10, 78A48

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*Corresponding author. *Email address:* `aftar@ukm.my` (M. A. A. Bakar)

1. Introduction

Several studies have promoted the wavelet transform (WT) as an alternative to spectral analysis (SA) for estimating the response amplitude operator (RAO) of linear systems [15, 29, 30]. The study of these systems is important because all structures are sensitive to vibration and some exploit this vibration. A stable linear system will respond to a stationary sinusoidal input at some specific frequency by vibrating at that frequency. However, the amplitude of the response relative to the amplitude of the input, known as the RAO or gain, depends on the frequency. There is also a phase shift, which depends on the frequency. Given this characterisation of a linear system, it is often more convenient to study the linear system in the frequency domain instead of in the time domain. Wavelet transforms have a potential advantage of displaying frequency composition over time. In contrast, the definition of a population spectrum as the Fourier transform of the autocovariance function is based on the assumption of a stationary random process.

The use of spectral analysis to estimate the RAO is justified for a general input rather than a stationary input, if the spectrum is considered as a sample estimate of a Fourier transform. A blow from an impact hammer, an accessory for spectrum analysers commonly used in model testing for lightweight structures, is a good example of a non-stationary input. Since spectral estimation of the RAO is not limited to a stationary input, it follows that the WT may not necessarily offer any advantage over the SA.

In this paper, we compare WT methods with SA methods for estimating the RAO of a linear system, with different classes of input signals and with different distributions of noise corrupting either input or response signals. The comparison is set in the context of a wave tank model of a heaving buoy wave energy device (HBWED). The reasons for this choice are that wave energy devices generally are receiving renewed attention as the need for renewable energy resources becomes increasingly apparent, and their response to the random wave environment is crucial for design. Specifically, the HBWED can plausibly be modelled as a single mode of the vibration system, and this allows a straightforward comparison of WT and SA estimation of the RAO. The oscillating wave surge converters (OWSC) are single degree of freedom devices — but the OWSC oscillates horizontally in surge instead of oscillating vertically in heave as the HBWED, and is nonlinear [32].

Spectral analysis has been used in the study of dynamic systems for many decades (e.g. see [14]), and the spectrum analyser has been standard equipment in test laboratories since the 1960s [13]. In contrast, although the Haar sequence was proposed in 1909 [10], the mathematical generalisation and the use of WT for the analysis of dynamic systems is still in the development stage [15, 21, 24]. Since a wavelet is localised in the time-scale domain, certain information can be accessed directly and immediately from the wavelet representation of a time series. This multiscale feature of wavelet transforms can be used to validate a dynamic model from a continuous wavelet transform of the process observations and model time series data [21].

Wavelets enable the detection of even very weak signals by using local amplification and compression, which has been advantageous in analysing dynamic systems. By using a wavelet transform, the random property of a chaotic response can also be observed