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Pricing Model for Convertible Bonds: A Mixed Fractional Brownian Motion with Jumps

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Abstract. A mathematical model to price convertible bonds involving mixed fractional Brownian motion with jumps is presented. We obtain a general pricing formula using the risk neutral pricing principle and quasi-conditional expectation. The sensitivity of the price to changing various parameters is discussed. Theoretical prices from our jump mixed fractional Brownian motion model are compared with the prices predicted by traditional models. An empirical study shows that our new model is more acceptable.

AMS subject classifications: 60J75, 60G22, 91G80

Key words: Mixed fractional Brownian motion, Poisson jump, convertible bond, empirical study.

1. Introduction

A convertible bond is a complex financial product that enables the holder to exchange the bond for the issuer's underlying stock in some specified circumstance. The characteristics of both bond and equity make the valuation of convertible bonds quite difficult. Indeed, the convertible bond trade is an emerging market, and it is important to consider more factors that influence convertible bond pricing than allowed for by existing pricing methods.

Theoretical research on convertible bond pricing was initiated by Ingersoll [1], who applied the well known Black-Scholes-Merton options pricing model. Following his work, Brennan & Schwartz [2] used corporate value as the basic variable to price convertible bonds, and then took into account the uncertainty inherent in interest rates and also the possibility of senior debt in the firm's capital structure [3]. Nyborg [4] considered the more complicated call and put features in convertible bond pricing under stochastic interest rates. In these ways, researchers have gradually added various factors to increase the accuracy of convertible bond pricing.

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The above articles all regard the price movement as a geometric Brownian motion. However, many empirical studies demonstrate that the distributions of the logarithmic returns on the financial asset usually exhibit self-similarity properties, heavy tails and long-range dependence in both auto-correlations and cross-correlations, and volatility clustering [5-10]. Indeed, the most common stochastic process that exhibits long-range dependence is a fractional Brownian motion (FBM). Furthermore, an FBM produces a burstiness in the sample path behaviour, an important aspect of financial time series. Consequently, it is natural to replace the Brownian motion with an FBM to make stochastic models more realistic [11-15].

Classical Itô theory cannot be applied to an FBM, and defining an associated proper stochastic integral is difficult [16]. A major difficulty is that an FBM is not semi-martingale, so to take the long memory property into account it is reasonable to prefer a mixed fractional Brownian motion (MFBM) in order to capture the price fluctuations of a financial asset [17-18]. An MFBM is essentially a family of Gaussian processes in a linear combination of Brownian motion and FBM — a class of long memory processes with the Hurst parameter $H \in (1/2, 1)$. The first work in economics using an MFBM is in Ref. [19], where for $H \in (3/4, 1)$ it was proven that an MFBM is equivalent in law to a Brownian motion, and hence financial markets driven by the MFBM is arbitrage-free. Recent additional applications have also been documented [20]. However, all of the above-mentioned earlier research considered that the logarithmic returns of the underlying stock are independent identically distributed normal random variables, whereas the empirical study of asset return indicates that discontinuities or jumps are an essential component of financial asset price series [21-23]. Merton [24] proposed a jump-diffusion process involving a Poisson jump, given the observed abnormal fluctuations in stock prices. Based on his theory, several authors have modelled the price of a convertible bond as a Brownian motion with Poisson jumps [25-27].

This article provides a theoretical, numerical and empirical contribution to the study of convertible bonds. To capture jumps or discontinuities and account for the long memory property, a combination of Poisson jumps and mixed fractional Brownian motion is used. As in Ref. [19], we assume $H \in (3/4, 1)$ throughout, an assumption validated by many previous empirical studies [20, 28]. Our jump mixed fractional Brownian motion (JMFBM) model produces empirically observed distributions of stock price changes that are skewed, leptokurtic, long memory and possess fatter tails than comparable normal distributions. A JMFBM model to price convertible bonds has not been investigated before, and our new model allows us to explore the sensitivities of the convertible bond price to changes in various relevant parameters. Numerical experiments supplemented by an empirical study indicate that our JMFBM model more closely predicts the actual market in convertible bonds than any purely Brownian motion model.

The rest of the article is organized as follows. Some MFBM results are recalled in Section 2, and we present the JMFBM pricing model for convertible bonds in Section 3. Section 4 contains the sensitivity analysis for the convertible bond price. We numerically compare our JMFBM model with traditional models in Section 5, and discuss the relationship between the convertible bond price and the parameters of the Hurst and jump. In