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Prediction-Correction Scheme for Decoupled Forward Backward Stochastic Differential Equations with Jumps

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Abstract. By introducing a new Gaussian process and a new compensated Poisson random measure, we propose an explicit prediction-correction scheme for solving decoupled forward backward stochastic differential equations with jumps (FBSDEJs). For this scheme, we first theoretically obtain a general error estimate result, which implies that the scheme is stable. Then using this result, we rigorously prove that the accuracy of the explicit scheme can be of second order. Finally, we carry out some numerical experiments to verify our theoretical results.

AMS subject classifications: 60H35, 65C20, 60H10

Key words: Prediction-correction scheme, decoupled forward backward stochastic differential equation with jumps, convergence analysis.

1. Introduction

In this article, we consider the numerical solution of decoupled *forward backward stochastic differential equations with jumps* (FBSDEJs). Applications of FBSDEJs can be found in many fields, including stochastic optimal control [4, 19, 27], risk measure [2, 24], nonlinear expectation [26], mathematical finance [7, 8], hedging [4, 12, 16], partial differential equations [3, 13, 17], and fractional differential equations [14, 23]. FBSDEJs can also be used to solve nonlocal diffusion problems [10, 11] — e.g. see Ref. [30]. Since it is usually very difficult to get solutions in closed explicit form, numerical methods are important for solving problems involving FBSDEJs in science and engineering. Compared with FBSDEs, for which many numerical schemes with high accuracy have been developed such as [6, 9, 31–34], there are relatively few numerical methods for solving FBSDEJs — cf. Refs. [1, 5, 35], however.

Standard *forward backward stochastic differential equations* (FBSDEs) [15, 18, 21] are driven by Brownian motion W_t, whereas FBSDEJs are driven by both the Brownian motion

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W_t and the compensated Poisson random measure $\tilde{\mu}(\cdot,\cdot)$. The existence and uniqueness of the solution of FBSDEJs have been studied by many researchers such as Tang & Li [27], Rong [25] and Wu [28, 29]. Barles *et al.* [3] proved that decoupled FBSDEJs have unique solutions, and established the relationship between FBSDEJs and a class of *partial-integral differential equations* (PIDEs). This relationship provides a new type of Feynman-Kac formula, which is a generalisation of one given by Pardoux & Peng [20].

Let $(\Omega,\mathscr{F},\mathbb{P},\{\mathscr{F}_t\}_{0\le t\le T})$ be a filtered, complete probability space with the filtration $\{\mathscr{F}_t\}_{0\le t\le T}$ generated by two mutually independent stochastic processes, the Brownian motion $W_t=(W_t^1,W_t^2,\cdots,W_t^d)^\tau$ and the Poisson random measure $\mu(A,t)$ on $E\times[0,T]$, where $(\cdot)^\tau$ denotes the transpose operator, $E=\mathbb{R}^q\backslash\{0\}$ with its Borel field $\mathscr{E}$, and λ is a σ-finite measure on $(E,\mathscr{E})$ such that $\int_E(1\wedge|e|^2)\lambda(de)<+\infty$. Suppose that the compensator of $\mu(A,t)$ is $\nu(de,dt)=\lambda(de)dt$. Then the corresponding compensated Poisson random measure is $\tilde{\mu}(de,dt):=\mu(de,dt)-\lambda(de)dt$. We consider the following decoupled FBSDEJs on the space $(\Omega,\mathscr{F},\mathbb{P},\{\mathscr{F}_t\}_{0\le t\le T})$ in integral form:

$$\begin{cases} X_t = X_0+\int_0^t b(s,X_s)ds+\int_0^t \sigma(s,X_s)dW_s \\ \qquad +\int_0^t\int_E c(s,X_{s-},e)\tilde{\mu}(de,ds), \quad \text{(SDEJ)} \\ Y_t = \xi+\int_t^T f(s,X_s,Y_s,Z_s,\Gamma_s)ds-\int_t^T Z_s dW_s \\ \qquad -\int_t^T\int_E U_s(e)\tilde{\mu}(de,ds), \quad \text{(BSDEJ)} \end{cases} \tag{1.1}$$

where the functions $b:[0,T]\times\mathbb{R}^q\to\mathbb{R}^q$, $\sigma:[0,T]\times\mathbb{R}^q\to\mathbb{R}^{q\times d}$, and $c:[0,T]\times\mathbb{R}^q\times E\to\mathbb{R}^q$ are referred to as the drift , diffusion and jump coefficients of the *forward stochastic differential equation with jumps* (SDEJ), respectively; $f:[0,T]\times\mathbb{R}^q\times\mathbb{R}^p\times\mathbb{R}^{p\times d}\times\mathbb{R}^p\to\mathbb{R}^p$ is called the generator of the *backward stochastic differential equation with jumps* (BSDEJ), and the process Γ_s is defined by $\Gamma_s=\int_E U_s(e)\eta(e)\lambda(de)$ for a given bounded function $\eta:E\to\mathbb{R}$, i.e., $\sup_{e\in E}|\eta(e)|<+\infty$. The terminal condition ξ, which could be an $\mathscr{F}_T$-measurable random vector in general cases, is assumed to be a deterministic function of X_T in this place, denoted by $\xi=\varphi(X_T)$. A quadruplet (X_t,Y_t,Z_t,U_t) is called an L^2-adapted solution to the decoupled FBSDEJs (1.1) if it is $\mathscr{F}_t$-adapted, square integrable and satisfies (1.1). The term "decoupled" means that the coefficient functions b,σ and c of the forward SDEJ do not depend on the solution (Y_t,Z_t,Γ_t) of the BSDEJ. In this work, we are aimed at numerically solving (X_t,Y_t,Z_t,Γ_t) instead of (X_t,Y_t,Z_t,U_t).

We introduce a new Gaussian stochastic process and a new compensated Poisson random measure, and propose a prediction-correction scheme for solving FBSDEJs using the trapezoidal rule, which is explicit for solving the unknowns. For this scheme, we theoretically obtain a general error estimate result that implies our explicit prediction-correction scheme is stable; and rigorously prove that its accuracy can be second-order, and numerical experiments confirm our theoretical results. In Section 2, we derive reference equations