East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.061116.230417a

Vol. **7**, No. 3, pp. 495-507
August 2017

JASMIN-based Two-dimensional Adaptive Combined Preconditioner for Radiation Diffusion Equations in Inertial Fusion Research

Xiaoqiang Yue[1], Xiaowen Xu[2] and Shi Shu[1,*]

[1] *School of Mathematics and Computational Science, Xiangtan University, Hunan 411105, China.*
[2] *Institute of Applied Physics and Computational Mathematics, Beijing 100088, China.*

Received 6 November 2016; Accepted 23 April 2017.

Abstract. We present a JASMIN-based two-dimensional parallel implementation of an adaptive combined preconditioner B_{co}^{α} for the solution of linear problems arising in the finite volume discretisation of one-group and multi-group radiation diffusion equations. We first propose the attribute of patch-correlation for cells of a two-dimensional monolayer piecewise rectangular structured grid without any suspensions based on the patch hierarchy of JASMIN, classify and reorder these cells via their attributes, and derive the conversion of cell-permutations. Using two cell-permutations, we then construct some parallel incomplete LU factorisation and substitution algorithms, to provide our parallel B_{co}^{α}-GMRES solver with the help of the default BoomerAMG in the HYPRE library. Numerical results demonstrate that our proposed parallel incomplete LU preconditioner (ILU) is of higher efficiency than the counterpart in the Euclid library, and that the proposed parallel B_{co}^{α}-GMRES solver is more robust and more efficient than the default BoomerAMG-GMRES solver.

AMS subject classifications: 65F10, 65N55, 65Y05, 65Z05

Key words: Radiation diffusion equations, combined preconditioner, adaptive technology, parallel computing, JASMIN.

1. Introduction

A common time-consuming bottleneck in numerical simulation of inertial confinement fusion (ICF) is the approximate solution of discretised algebraic systems arising from multi-group radiation diffusion (MGD) equations, since they invariably suffer from ill-conditioned coefficient matrices [1, 2]. The MGD equations involve non-matching meshes with large deformations, multiple spatial and temporal scales, very complicated couplings among

*Corresponding author. *Email address:* `shushi@xtu.edu.cn` (S. Shu)

dozens of physical quantities that are strongly nonlinear and discontinuous in a multi-component medium [3], and ultra-large scale algebraic systems for higher resolution ICF simulations [4].

Preconditioned Krylov subspace methods are most frequently chosen to manage the ill-conditioned algebraic systems. However, computational efficiency and robustness are severely affected by the main existing standalone preconditioners for solving the MGD equations [5–9] — viz. the incomplete LU preconditioner (ILU) [10,11], the algebraic multigrid (AMG), domain decomposition and multilevel preconditioners [12–17]. There has recently been an increased interest in designing combined preconditioners [2,18], which can effectively manage the complex coupling relationships in the MGD equations. It is notable that the adaptive combined preconditioner B_{co}^{α} proposed in Ref. [2] can improve consistency with respect to different matrices arising from nonlinear Picard iterations in quite a number of integration time-steps. The successful Eulerian and Lagrangian applications of B_{co}^{α} in simulating hydrodynamic instabilities by LARED-S [1] strongly suggested the pursuit of some hybrid MPI/OpenMP parallel B_{co}^{α}-GMRES solver to us.

Developed by the Institute of Applied Physics and Computational Mathematics, the JASMIN (J Adaptive Structured Meshes applications INfrastructure) is a parallel application framework [19] suitable for large-scale problems that involve adaptive structured or unstructured joint structured mesh, providing powerful parallel performance on massively parallel machines [1, 20–22]. Since we mainly focus on the linear solver, only a brief description of the relevant underlying numerical techniques is given here — more details about JASMIN can be found in Ref. [19]. The structured adaptive mesh refinement (SAMR) mesh used in JASMIN consists of a hierarchy of refinement levels. A fine level is constructed dynamically by refining the local region on a coarser level with a refinement ratio r — e.g. $r = 2$ or 4. Each level is uniform and organised as a union of rectangular patches, with the refinement criteria supplied by users but the management of the mesh data structure encapsulated in JASMIN. Thus in ICF code based on JASMIN, the time integration is performed on the SAMR mesh via a refinement strategy [23, 24] where the time-stepping advances from the coarsest to the finest level individually, followed by a synchronisation process performed from the finest to the coarsest level. The most important feature of the aforementioned refinement time-stepping strategy is that the designs and parallelisms of the solvers are only applicable for each level. In order to reduce the time-to-solution, a JASMIN-based parallel B_{co}^{α}-GMRES on single level is therefore necessary for the radiation diffusion solution in ICF simulations.

We propose a massively parallel ILU(0) preconditioner on the monolayer piecewise rectangular structured grid without any suspensions, fabricate the parallel B_{co}^{α} by invoking the BoomerAMG [25] provided in the HYPRE library, and apply B_{co}^{α}-GMRES to the classical MGD equations. First of all, we characterise the attribute of patch-correlation for every grid cell based on the patch hierarchy of JASMIN, classify these grid cells via attributes, and reorder them as $\mathscr{C}_i$ by classifications. This can guarantee that all of the grid cells are entirely unrelated to those with the same attribute in other processors, resulting in a significant reduction in data-communication. Furthermore, we originate the conversion between $\mathscr{C}_i$ and the lexicographical order $\mathscr{D}_i$ in JASMIN to interface practical applications smoothly.